Statistical ensemble as a dynamic system

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Abstract

It is shown that description of a stochastic particle in the framework of of axiomatic quantum mechanics is not complete. Description of a stochastic particle in framework of classical gas dynamics is more complete.

Statistical ensemble is a tool for statistical description of indeterministic or partially indeterministic processes. Concept of statistical ensemble is slightly different in various branches of physics. Usually a statistical ensemble is a superstructure over dynamics. The dynamics can be classical or quantum. In both cases the statistical ensemble is a superstructure over dynamics. But a statistical ensemble is not defined as an object of dynamics. Ludwig Boltzmann was the first researcher, who used the statistical ensemble as a dynamic system for description of stochastic motion of gas molecules. But in this case he did not used the term statistical ensemble.

We define statistical ensemble as a dynamic system, consisting of many identical dynamic systems $\mathcal{S}$. For instance, gas, consisting of identical molecules, is a dynamic system. The gas is a statistical ensemble of identical molecules. If collision of gas molecules absent, the molecule motion is deterministic, and one can derive dynamic equation for motion of a single molecule from gas dynamic equation. If there are collisions between molecules, the molecule motion is stochastic (indeterministic). In this case there are no dynamic equations for a single molecule. However, the mean motion of a single molecule can be derived from the gas dynamic equations. In other words, gas as a dynamic system admits one to obtain some statistical information on a motion of a single molecule. Investigating properties of the molecular collision, which is responsible for the stochastic motion of gas molecules, Boltzmann obtained a kinetic equation, which describes evolution of the velocity distribution. In other words, Boltzmann has obtained statistical description of stochastic motion of a single molecule, although dynamic equations for stochastic motion of a single molecule did not exist. In the given case, the gas plays the role of statistical ensemble $\mathcal{E}[\mathcal{S}]$, where $\mathcal{S}$ is a single molecule.

The Boltzmann's papers has been underestimated by scientific community, which considered them as papers on fluid dynamics, whereas in reality they were papers on statistical description of the indeterministic particle motion. It was especially clear, when it appears, that quantum particles can be described by means of the gas dynamics, where molecules interact via some force field κ instead of collision.

If we consider a set of many noninteracting identical particles, the statistical ensemble $\mathcal{E}[\mathcal{S}]$ (gas) is described by the action

$$\mathcal{A}[x] = -\int mc\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d\tau d\boldsymbol{\xi}, \quad \dot{x}^i \equiv \frac{\partial x^i(\tau,\boldsymbol{\xi})}{\partial \tau} \tag{1}$$

where $x = \left(x^0(\tau,\boldsymbol{\xi}), x^1(\tau,\boldsymbol{\xi}), x^2(\tau,\boldsymbol{\xi}), x^3(\tau,\boldsymbol{\xi})\right)$, $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$ is a label of a particle (Lagrangian coordinates), τ is a parameter along the world line of the particle, m is the particle mass and c is the speed of the light.

If the particles interact via some field $\kappa^i = \left(\kappa^0(x), \kappa^1(x), \kappa^2(x), \kappa^3(x)\right)$, which changes the particle mass

$$m \rightarrow M = m\sqrt{1 + \lambda^2(\kappa_i\kappa^i + \partial_i\kappa^i)}, \quad \partial_i \equiv \frac{\partial}{\partial x^i} \tag{2}$$

The action takes the form

$$\mathcal{A}[x,\kappa] = -\int mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d\tau d\boldsymbol{\xi}, \tag{3}$$

$$K = \frac{M}{m} = \sqrt{1 + \lambda^2(\kappa_i\kappa^i + \partial_i\kappa^i)} \tag{4}$$

Dynamic equations for variables x and κ are derived from (3) by means of variation of (3) with respect to x^i and κ^i respectively.

Variation of (3) leads to dynamic equation

$$-\hbar^2\partial_k\partial^k\psi - \left(m^2c^2 + \frac{\hbar^2}{4}(\partial_l s_\alpha)(\partial^l s_\alpha)\right)\psi = -\hbar^2\frac{\partial_l(\rho\partial^l s_\alpha)}{2\rho}(\sigma_\alpha - s_\alpha)\psi \tag{5}$$

where 3-vector $\mathbf{s} = \{s_1, s_2, s_3,\}$ is defined by the relations

$$\rho = \psi^*\psi, \qquad s_\alpha = \frac{\psi^*\sigma_\alpha\psi}{\rho}, \qquad \alpha = 1,2,3 \tag{6}$$

$$\psi = \binom{\psi_1}{\psi_2}, \qquad \psi^* = (\psi_1^*, \psi_2^*), \tag{7}$$

and $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ are 2×2 Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{8}$$

The wave function ψ has been introduced, because the wave function is a natural attribute of fluid dynamics [1].

In the case of nonrotational flow the wave function is one component the dynamic equation (8) turns to the Klein-Gordon equation

$$-\hbar^2\partial_k\partial^k\psi - m^2c^2\psi = 0 \tag{9}$$

Details of equation (5) derivation can be found in [2].

Connection between the gas dynamics and quantum equations was known long ago [3, 4]. However, this connection was one-sided. One can derive fluid dynamic equations from the Schrödinger equation, but one was not able to derive the Schrödinger equation from the fluid dynamics. In this relation derivation of Klein-Gordon equation as a gas dynamic equation looks as inverse operation.

Thus, quantum dynamic equation can be derived in the framework of classical dynamics without a use of quantum principles. It is a very unexpected result, because it is used to think, that the Klein-Gordon equations can be obtained only by a use of quantum principles in the framework of axiomatic quantum theory. The Klein-Gordon equation (9) is simply classical gas dynamic equation for the gas, where interaction between molecules is described by relation (2). It means that quantum phenomena may be explained in the framework classical dynamics, if one chooses a proper interaction field between particles.

Besides, the example of Boltzmann consideration shows, that description of the particle stochasticity by means of gas dynamic equation is incomplete. Investigation of the field, which is responsible for molecular interaction (collision), admits one to obtain a more complete description of the molecular motion stochasticity in terms of kinetic equation. The Klein-Gordon equation (9) is a kind of the classical gas dynamic equation. One can expect that investigation of κ-field (2) admits one to obtain a more complete description of the quantum stochasticity.

In the axiomatic quantum theory, where the quantum stochasticity is determined by the quantum principles, the Klein-Gordon equation (9) realizes maximally possible description of the particle stochasticity. Description of quantum particles in terms of classical dynamics, where some classical force field is responsible for quantum stochasticity, generates a new direction in the elementary particle theory. Elementary particles have some internal structure in the framework of this direction. They are not pointlike objects, as it takes place in the axiomatic quantum theory. Existence of the κ-field (2) is conditioned by discrete space-time geometry, and quantum constant $\hbar$ is connected with minimal length λ_0 of the discrete space-time geometry. Investigation of the κ-field generated structural direction in the elementary particle theory [5], which admits one to investigate internal structure of elementary particles.

It is surprising, how a change of the statistical ensemble definition generates essential change of the elementary particle theory.

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