Non-Euclidean method of the generalized geometry construction and its application to space-time geometry.

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Abstract

Non-Euclidean method of the generalized geometry construction is considered. According to this approach any generalized geometry is obtained as a result of deformation of the proper Euclidean geometry. The method may be applied for construction of space-time geometries. Uniform isotropic space-time geometry other, than that of Minkowski, is considered as an example. The problem of the geometrical objects existence and their temporal evolution may be considered in the constructed space-time geometry. Such a statement of the problem is impossible in the framework of the Riemannian space-time geometry. Existence and dynamics of microparticles is considered to be conditioned by existence of corresponding geometrical objects and their temporal evolution in the space-time. Geometrization of the particle mass and its momentum is produced.

1 Introduction

Any (generalized) geometry is a set of propositions on properties of geometrical objects. Geometrical object is a subset of points of the point set Ω, where the geometry is given. The number of these propositions is very large, and labelling of these propositions by real numbers is rather difficult, because the capacity of the set of real numbers is not sufficient for such a labelling. Labelling by means of functions appears to be more effective, because the set of functions is "more powerful", than the set of real numbers. For instance, let f be an integer function of integer argument x. Let $f \in [0, M-1]$ and $x \in [0, N-1]$, where M and N are natural numbers. Then the number N_f of all functions f is $N_f = M^N$. If f is a function of two integer arguments $x_1 \in [0, N-1]$ and $x_2 \in [0, N-1]$, then the number of functions $N_f = M^{\left(N^2\right)}$. In other words, the number of functions N_f increases much faster, then the number of values N of the function arguments and the number M of the function values. Thus, labelling of the geometry propositions by means of functions seems to be more effective, than labelling by means of numbers.

However, investigating functions of real variables, nobody tries to calculate the number of these functions. This calculation is used only for Boolean functions of Boolean arguments. In this case $M = 2$, $N = 2$ and $N_f = 2^2 = 4$. In the case of Boolean functions of two arguments, we have $N_f = 2^4 = 16$. The functions of real variables are described and investigated by subsequent combinations of simple algorithms such as: summation, multiplication, raising to a power, taking logarithm, etc.

T-geometry is a geometry, which is constructed by means of the proper Euclidean geometry deformation. Any geometry is a construction, which describes the mutual disposition of points and geometrical objects on the point set Ω. The mutual disposition of points $P, Q \in \Omega$ is described by the distance $\rho(P, Q)$ between any two points $P, Q \in \Omega$. In the space-time geometry the distance is real and positive for some pairs of points, and it is imaginary for some other pairs of points. It is more convenient and useful to use the function $\sigma(P, Q) = \frac{1}{2}\rho^2(P, Q)$, which is real for any pair of points. The function $\sigma(P, Q)$ is known as the world function [1]. We shall use the world function for description of the mutual disposition of points of the point set Ω.

One should expect, that giving the world function

$$\sigma: \qquad \Omega \times \Omega \to \mathbb{R}, \qquad \sigma(P, Q) = \sigma(Q, P), \qquad \sigma(P, P) = 0, \qquad \forall P, Q \in \Omega \tag{1.1}$$

for all pairs P, Q of points on the point set Ω, we determine the geometry completely. Let us show this for the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, described by the Euclidean world function σ_{E}. Thereafter we expand this result to any geometry $\mathcal{G}$, described by arbitrary world function σ, satisfying the relations (1.1).

Contemporary presentation of the proper Euclidean geometry is based on the concept of the linear vector space V_n equipped with the scalar product of any two vectors, given on the linear vector space. Here index n means the dimension of the linear vector space, which is defined as the maximal number of linear independent vectors. The Euclidean n-dimensional point space E_n is obtained from the n-dimensional vector space V_n, if one considers all those vectors, whose origins coincide. Then the set of all ends of all vectors forms the Euclidean point space $\Omega = \mathbb{R}^n$. Any two points P, Q form the vector $\mathbf{PQ} \equiv \overrightarrow{PQ}$, belonging to the vector space $V_n = \mathbb{R}^n$.

The vector $\mathbf{PQ} \equiv \overrightarrow{PQ}$ is the ordered set of two points $\{P, Q\}$, $P, Q \in \mathbb{R}^n$. The length $|\mathbf{PQ}|_{\mathrm{E}}$ of the vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{PQ}|_{\mathrm{E}}^2 = 2\sigma_{\mathrm{E}}(P, Q) \tag{1.2}$$

where index "E" means that the length of the vector is taken in the proper Euclidean space.

The scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)_{\mathrm{E}}$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{P}_2$ having the common origin P_0 is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)_{\mathrm{E}} = \sigma(P_0, P_1)_{\mathrm{E}} + \sigma(P_0, P_2)_{\mathrm{E}} - \sigma(P_1, P_2)_{\mathrm{E}} \tag{1.3}$$

which is obtained from the Euclidean relation

$$|\mathbf{P}_1\mathbf{P}_2|_{\mathrm{E}}^2 = |\mathbf{P}_0\mathbf{P}_2 - \mathbf{P}_0\mathbf{P}_1|_{\mathrm{E}}^2 = |\mathbf{P}_0\mathbf{P}_2|_{\mathrm{E}}^2 + |\mathbf{P}_0\mathbf{P}_1|_{\mathrm{E}}^2 - 2(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)_{\mathrm{E}} \tag{1.4}$$

by means of the relation (1.2). In particular

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_1)_{\mathrm{E}} = 2\sigma_{\mathrm{E}}(P_0, P_1) = |\mathbf{P}_0\mathbf{P}_1|_{\mathrm{E}}^2 \tag{1.5}$$

Note that the relation (1.3) is the definition of the scalar product via the Euclidean world function σ_{E}, whereas in the conception of the linear vector space the relation (1.4) is so called cosine theorem.

In the proper Euclidean space one can define the scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}}$ of two remote vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$. It is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} = \sigma(P_0, Q_1)_{\mathrm{E}} + \sigma(P_1, Q_0)_{\mathrm{E}} - \sigma(P_0, Q_0)_{\mathrm{E}} - \sigma(P_1, Q_1)_{\mathrm{E}} \tag{1.6}$$

which follows from evident Euclidean relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} = (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}_1)_{\mathrm{E}} - (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}_0)_{\mathrm{E}} \tag{1.7}$$

and relation (1.3), written for two terms in rhs of (1.7).

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}_1)_{\mathrm{E}} = \sigma(P_0, P_1)_{\mathrm{E}} + \sigma(P_0, Q_1)_{\mathrm{E}} - \sigma(P_1, Q_1)_{\mathrm{E}}$$

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}_0)_{\mathrm{E}} = \sigma(P_0, P_1)_{\mathrm{E}} + \sigma(P_0, Q_0)_{\mathrm{E}} - \sigma(P_1, Q_0)_{\mathrm{E}}$$

The necessary and sufficient condition of linear dependence of n vectors ,$\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,...$\mathbf{P}_0\mathbf{P}_n$, defined by $n+1$ points $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\}$ in the proper Euclidean space, is a vanishing of the Gram's determinant

$$F_n(\mathcal{P}^n) \equiv \det ||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)_{\mathrm{E}}||, \qquad i, k = 1, 2, ...n \tag{1.8}$$

Expressing the scalar products $(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)_{\mathrm{E}}$ in (1.8) via world function σ_{E} by means of relation (1.3), we obtain definition of linear dependence of n vectors ,$\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,...$\mathbf{P}_0\mathbf{P}_n$ in the proper Euclidean space in the form

$$F_n(\mathcal{P}^n) = 0 \tag{1.9}$$

$$F_n(\mathcal{P}^n) \equiv \det ||\sigma(P_0, P_i)_{\mathrm{E}} + \sigma(P_0, P_k)_{\mathrm{E}} - \sigma(P_i, P_k)_{\mathrm{E}}||, \qquad i, k = 1, 2, ...n \tag{1.10}$$

Relations (1.9), (1.10) form σ-immanent definition (i.e. the definition in terms of the world function) of the linear dependence. This definition is obtained from the theorem on the condition of the linear dependence of n vectors in the proper

Euclidean space. This definition does not contain any reference to the linear space. It looks as a linear dependence without a linear space.

In particular, two vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{Q}_0\mathbf{Q}_1$ are collinear (linear dependent) $\mathbf{P}_0\mathbf{P}_1 \parallel \mathbf{Q}_0\mathbf{Q}_1$, if

$$\mathbf{P}_0\mathbf{P}_1 \parallel \mathbf{Q}_0\mathbf{Q}_1 : \qquad \left\| \begin{array}{cc} |\mathbf{P}_0\mathbf{P}_1|_{\mathrm{E}}^2 & (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} \\ (\mathbf{Q}_0\mathbf{Q}_1.\mathbf{P}_0\mathbf{P}_1)_{\mathrm{E}} & |\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{E}}^2 \end{array} \right\| = 0 \tag{1.11}$$

Two vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{Q}_0\mathbf{Q}_1$ are in parallel, if

$$\mathbf{P}_0\mathbf{P}_1 \upuparrows_{\mathrm{E}} \mathbf{Q}_0\mathbf{Q}_1 : \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} = |\mathbf{P}_0\mathbf{P}_1|_{\mathrm{E}} \cdot |\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{E}} \tag{1.12}$$

Two vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{Q}_0\mathbf{Q}_1$ are antiparallel, if

$$\mathbf{P}_0\mathbf{P}_1 \uparrow\downarrow_{\mathrm{E}} \mathbf{Q}_0\mathbf{Q}_1 : \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} = -\,|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{E}} \cdot |\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{E}} \tag{1.13}$$

Index "E" means that the scalar product and parallelism are considered in the proper Euclidean space.

Now we can define equivalence of two vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{Q}_0\mathbf{Q}_1$ in the σ-immanent form. (i.e. in terms of the world function σ). Two vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent (equal), if

$$\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1 : \qquad (\mathbf{P}_0\mathbf{P}_1 \upuparrows \mathbf{Q}_0\mathbf{Q}_1) \wedge (|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|) \tag{1.14}$$

or

$$\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1 \quad : \qquad ((\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1|) \tag{1.15}$$

$$\wedge\, (|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|) \tag{1.16}$$

The property of the equivalence of two vectors in the proper Euclidean geometry is reversible and transitive. It means

$$\text{if} \quad \mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1, \quad \text{then} \quad \mathbf{Q}_0\mathbf{Q}_1\mathrm{eqv}\mathbf{P}_0\mathbf{P}_1 \tag{1.17}$$

$$\text{if} \quad (\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1) \wedge (\mathbf{Q}_0\mathbf{Q}_1\mathrm{eqv}\mathbf{R}_0\mathbf{R}_1)\,, \quad \text{then} \quad \mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{R}_0\mathbf{R}_1 \tag{1.18}$$

However, the equivalence is reversible and transitive only in the proper Euclidean geometry, where the property of parallelism of two vectors is reversible and transitive. In the arbitrary generalized geometry the property of parallelism as well as the equivalence are reversible and intransitive, in general. Intransitivity of the equivalence property is connected with its multivariance, when there are many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$, $\mathbf{Q}_0\mathbf{Q}_1''$,...which are equivalent to the vector $\mathbf{P}_0\mathbf{P}_1$, but not equivalent between themselves. Multivariance of the equivalence property is conditioned by the fact, that equations (1.15), (1.16), considered as a system of equations for determination of the point Q_1 (at fixed points P_0, P_1, Q_0) has, many solutions, in general. It is possible also such a situation, when equations (1.15), (1.16) have no solution. Thus, the equivalence is multivariant and intransitive, in general.

In the proper Euclidean geometry the system of equations (1.15), (1.16) has always one and only one solution. In this case the property of equivalence is single-variant, and transitive. However, already in the Minkowski space-time geometry the equivalence of only timelike vectors is single-variant and transitive. Equivalence of spacelike vectors is multivariant and intransitive in the Minkowski space-time geometry. However, nobody pays attention to this fact, because the spacelike vectors are not used practically in applications to physics and to mechanics.

We shall distinguish between the equality relation (=) and the equivalence relation (eqv), because the equality relation is always single-variant and transitive, whereas the equivalence relation is multivariant and intransitive, in general.

The sum $\mathbf{P}_0\mathbf{P}_2$ of two vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$

$$\mathbf{P}_0\mathbf{P}_2 = \mathbf{P}_0\mathbf{P}_1 + \mathbf{P}_1\mathbf{P}_2$$

may be defined only in the case, when the end P_1 of the vector $\mathbf{P}_0\mathbf{P}_1$ coincide with the origin P_1 of the vector $\mathbf{P}_1\mathbf{P}_2$. However, using concept of equivalence, we may define the sum of two arbitrary vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ as a vector $\mathbf{R}_0\mathbf{R}_2$ with the origin at the point R_0 by means of the relation

$$\mathbf{R}_0\mathbf{R}_2 = (\mathbf{R}_0\mathbf{R}_1 + \mathbf{R}_1\mathbf{R}_2)\,\mathrm{eqv}\,(\mathbf{P}_0\mathbf{P}_1 + \mathbf{Q}_0\mathbf{Q}_1) \tag{1.19}$$

where vectors $\mathbf{R}_0\mathbf{R}_1$ and $\mathbf{R}_1\mathbf{R}_2$ are defined by the relations

$$\mathbf{R}_0\mathbf{R}_1\mathrm{eqv}\mathbf{P}_0\mathbf{P}_1, \qquad \mathbf{R}_1\mathbf{R}_2\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1 \tag{1.20}$$

However, the sum $\mathbf{R}_0\mathbf{R}_2$ appears to be multivariant, because of multivariance of relations (1.20). Besides, the sum (1.19) depends, in general, on the order of terms in the sum, because instead of (1.20) one may use the relations

$$\mathbf{R}_0\mathbf{R}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1, \qquad \mathbf{R}_1\mathbf{R}_2\mathrm{eqv}\mathbf{P}_0\mathbf{P}_1 \tag{1.21}$$

Multiplication of the vector $\mathbf{Q}_0\mathbf{Q}_1$ by the real number α is defined as follows. Vector $\mathbf{P}_0\mathbf{P}_1$ is the result of multiplication of the vector $\mathbf{Q}_0\mathbf{Q}_1$ by the real number α, if

$$\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\,(\alpha\mathbf{Q}_0\mathbf{Q}_1): \qquad \left\{ \begin{array}{ll} (\mathbf{P}_0\mathbf{P}_1 \upuparrows \mathbf{Q}_0\mathbf{Q}_1) \wedge (|\mathbf{P}_0\mathbf{P}_1| = |\alpha|\,|\mathbf{Q}_0\mathbf{Q}_1|)\,, & \text{if} \;\; \alpha \geq 0 \\ (\mathbf{P}_0\mathbf{P}_1 \uparrow\downarrow \mathbf{Q}_0\mathbf{Q}_1) \wedge (|\mathbf{P}_0\mathbf{P}_1| = |\alpha|\,|\mathbf{Q}_0\mathbf{Q}_1|)\,, & \text{if} \;\; \alpha < 0 \end{array} \right. \tag{1.22}$$

It is possible another version of multiplication by the real number α, which distinguishes from (1.22) only for $\alpha = 0$

$$\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\,(\alpha\mathbf{Q}_0\mathbf{Q}_1): \qquad \left\{ \begin{array}{ll} (\mathbf{P}_0\mathbf{P}_1 \upuparrows \mathbf{Q}_0\mathbf{Q}_1) \wedge (|\mathbf{P}_0\mathbf{P}_1| = |\alpha|\,|\mathbf{Q}_0\mathbf{Q}_1|)\,, & \text{if} \;\; \alpha > 0 \\ \mathbf{P}_0\mathbf{P}_0, \;\; \text{if} \;\; \alpha = 0 & \\ (\mathbf{P}_0\mathbf{P}_1 \uparrow\downarrow \mathbf{Q}_0\mathbf{Q}_1) \wedge (|\mathbf{P}_0\mathbf{P}_1| = |\alpha|\,|\mathbf{Q}_0\mathbf{Q}_1|)\,, & \text{if} \;\; \alpha < 0 \end{array} \right. \tag{1.23}$$

For the proper Euclidean geometry both versions coincide.

To complete the σ-immanent description of the proper Euclidean space, one needs to determine properties of the Euclidean world function $\sigma_{\rm E}$. They are presented in the second section, where one can see, that the specific properties are different for Euclidean spaces of different dimensions.

Presentation of the Euclidean geometry in the σ-immanent form (in terms of the world function $\sigma_{\rm E}$) admits one to use non-Euclidean method of the generalized geometry construction. The Euclidean method of the geometry construction is based on derivation of the geometrical propositions (theorems) from primordial propositions (axioms) of the constructed geometry by means of logical reasonings and mathematical calculations. This method is used always at the generalized geometry construction. It reminds construction of functions of real variables by means of simple procedures: summation, multiplication, etc.

The main defect of the Euclidean method is a necessity of a test of the primordial axioms consistency. Such a test is a very complicated procedure, which has been produced only for the proper Euclidean geometry. Besides, the primordial axioms are comparatively simple only for uniform generalized geometries. In this case the set of axioms is the same for all space regions, described by the generalized geometry. In the case of non-uniform geometry the set of axioms is different for different space regions.

As far as there was only Euclidean method of the geometry construction, a tendency appeared to prescribe the properties of the Euclidean method to the geometry itself. As far as usually the geometry was constructed, starting from a system of axioms, the tendency appeared to consider any system of axioms, which contains concepts of point and of straight line, as a kind of geometry (for instance, projective geometry, affine geometry, etc.). In reality the Euclidean method of the geometry construction and the system of axioms are something external with respect to the Euclidean geometry in itself, as well as to other generalized geometries (for instance, to the Riemannian geometry). Unfortunately, some mathematicians could not separate the method of the geometry construction from the geometry in itself, and this circumstance was a reason of rejection of the geometry, constructed by the non-Euclidean method [2].

We suggest a non-Euclidean method of the generalized geometry construction. This method may be considered as a construction of the generalized geometry by means of a deformation of the proper Euclidean geometry, when the Euclidean world function $\sigma_{\rm E}$ is replaced by the world function σ of the geometry in question in all propositions of the Euclidean geometry. As far as all propositions of the Euclidean geometry may be labelled by the Euclidean world function $\sigma_{\rm E}$, which describes each of such propositions completely, the replacement $\sigma_{\rm E}\rightarrow\sigma$ in all these propositions leads to a construction of the generalized geometry, described by the world function σ.

The non-Euclidean method of the geometry construction can be carried out, if we have the proper Euclidean geometry in the σ-immanent form. At this method one does not need to separate primordial axioms. One does not need to test their compatibility and to deduce other geometrical propositions. At this approach all propositions of the proper Euclidean geometry have equal rights. This approach reminds labelling of Boolean functions, which appears to be possible, because of small number of the Boolean functions. In the given case the labelling appears to be possible, because the proper Euclidean geometry has been constructed and presented in the σ-immanent form.

There is another analogy between the Boolean functions and the σimmanent presentation of the Euclidean geometry. The Boolean functions form the mathematical tool of the formal logic. The formalism of the σ-immanent description forms a mathematical tool of the "geometric logic", i.e. a system of rules for construction of any generalized geometry [3]. Maybe, on may speak about some metageometry which deals with all possible geometry simultaneously.

Unexpected feature of the "geometric logic" is a multivariance of operations in this logic. All operations of the conventional formal logic are single-variant. It is convenient and customary, however, not all generalized geometry can be constructed on the basis of single-variant logic rules. Besides, the real space-time geometry is multivariant, and it is very important to have a possibility of working with a multivariant "geometric logic". Multivariance appears to be a very general property of the generalized geometries and, in particular, of the space-time geometry.

The generalized geometry, constructed by means of a deformation of the proper Euclidean geometry is called T-geometry (tubular geometry), because in T-geometry straight lines are, in general, surfaces (tubes), but not one-dimensional lines. This fact is conditioned by the multivariant character of the parallelism in T-geometry. Indeed, the straight line $\mathcal{T}_{P_0P_1}$, passing through points P_0, P_1 is defined by the relation

$$\mathcal{T}_{P_0P_1} = \{R|\mathbf{P}_0\mathbf{P}_1||\mathbf{P}_0\mathbf{R}\} \tag{1.24}$$

and collinearity of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{R}$ is determined by one equation (1.11)

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{R})^2 = |\mathbf{P}_0\mathbf{P}_1|^2 \cdot |\mathbf{P}_0\mathbf{R}|^2 \tag{1.25}$$

In the n-dimensional space one equation (1.25) determines, in general, $(n-1)$-dimensional surface. If the world function deviates from the Euclidean world function slightly, this surface looks as a tube.

T-geometry is interesting by its application to physics, in particular, to the space-time geometry and dynamics. In T-geometry one can set the question on existence of geometrical objects. In the Minkowski space-time geometry the question on existence of a geometrical object is trivial in the sense, that any geometrical object (any subset $\mathcal{O}$ of points of the point set Ω) may be considered as existing.

Let the set of points $\mathcal{O}_{P_0P_1...P_n} = \mathcal{O}(\mathcal{P}^n)$ be a geometrical object , where $\mathcal{P}^n \equiv \{P_0, P_1, ...P_n\}$ are $n+1$ characteristic points, determining the geometrical object $\mathcal{O}(\mathcal{P}^n)$. The problem of existence of the geometrical object $\mathcal{O}(\mathcal{P}^n)$ is formulated as follows. The geometrical object $\mathcal{O}(\mathcal{P}^n)$ exists at the point $P_0 \in \Omega$, if at any point $Q_0 \in \Omega$ one can construct such a subset of points $\mathcal{O}(\mathcal{Q}^n)$, $\mathcal{Q}^n \equiv \{Q_0, Q_1, ...Q_n\}$, that $n(n+1)/2$ relations take place

$$\mathbf{P}_i\mathbf{P}_k\text{eqv}\mathbf{Q}_i\mathbf{Q}_k, \qquad i,k = 0,1,...n, \qquad i<k \tag{1.26}$$

According to (1.14) equivalence $\mathbf{P}_i\mathbf{P}_k\mathrm{eqv}\mathbf{Q}_i\mathbf{Q}_k$ means two relations

$$(\mathbf{P}_i\mathbf{P}_k.\mathbf{Q}_i\mathbf{Q}_k) = |\mathbf{P}_i\mathbf{P}_k| \cdot |\mathbf{Q}_i\mathbf{Q}_k|, \qquad |\mathbf{P}_i\mathbf{P}_k| = |\mathbf{Q}_i\mathbf{Q}_k| \tag{1.27}$$

Thus, relations (1.26) form the system of $n(n+1)$ equations for determination of $4n$ coordinates of points $Q_1, Q_2, ...Q_n$ in the 4-dimensional space-time. Coordinates of the point Q_0 are given, because they determine the displacement of the object $\mathcal{O}(\mathcal{Q}^n)$. In the Minkowski space-time it follows from $(\mathbf{P}_0\mathbf{P}_k\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_k) \wedge (\mathbf{P}_0\mathbf{P}_i\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_i)$ that $\mathbf{P}_k\mathbf{P}_i\mathrm{eqv}\mathbf{Q}_k\mathbf{Q}_i$, provided all vectors are timelike. It means that not all equations (1.26) are independent. Instead of $n(n+1)$ equations (1.27) we have $2n$ relations

$$(\mathbf{P}_0\mathbf{P}_k.\mathbf{Q}_0\mathbf{Q}_k) = |\mathbf{P}_0\mathbf{P}_k| \cdot |\mathbf{Q}_0\mathbf{Q}_k|, \qquad |\mathbf{P}_0\mathbf{P}_k| = |\mathbf{Q}_0\mathbf{Q}_k|, \qquad k = 1, 2, ...n \tag{1.28}$$

for determination of $4n$ coordinates of points $Q_0, Q_1, ...Q_n$.

The structure of the relations (1.28) in the Minkowski space-time is such, that two relations

$$(\mathbf{P}_0\mathbf{P}_k.\mathbf{Q}_0\mathbf{Q}_k) = |\mathbf{P}_0\mathbf{P}_k| \cdot |\mathbf{Q}_0\mathbf{Q}_k|, \qquad |\mathbf{P}_0\mathbf{P}_k| = |\mathbf{Q}_0\mathbf{Q}_k| \tag{1.29}$$

determine uniquely four coordinates of the point Q_k, provided the vector $\mathbf{P}_0\mathbf{P}_k$ is timelike, i.e. $|\mathbf{P}_0\mathbf{P}_k|^2 > 0$.

In other uniform isotropic space-times the structure of relations (1.29) has another character. In this case two relations (1.29) do not determine uniquely four coordinates of the point Q_k. Besides, the relation $\mathbf{P}_k\mathbf{P}_i\mathrm{eqv}\mathbf{Q}_k\mathbf{Q}_i$ is not a corollary of $(\mathbf{P}_0\mathbf{P}_k\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_k) \wedge (\mathbf{P}_0\mathbf{P}_i\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_i)$ and relations (1.26) form $n(n+1)$ relations which are independent, in general. For two characteristic points Q_0, Q_1 we have $n = 1$ and the number of equations $n(n+1) = 2$ is less, than the number of coordinates $4n = 4$ of point Q_1.

In the case of three characteristic points Q_0, Q_1, Q_2 we have $n = 2$, and the number of equations $n(n+1) = 6$ is less, than the number of coordinates $4n = 8$ of points Q_1, Q_2

In the case of four characteristic points Q_0, Q_1, Q_2, Q_3 we have $n = 3$, and the number of equations $n(n+1) = 12$ is equal to the number of coordinates $4n = 12$ of points Q_1, Q_2, Q_3.

Finally, in the case of five characteristic points Q_0, Q_1, Q_2, Q_3, Q_4 we have $n = 4$, and the number of equations $n(n+1) = 20$ is more than the number of coordinates $4n = 16$ of points Q_1, Q_2, Q_3, Q_4.

It means, that geometrical objects, having more, than four characteristic points do not exist in the multivariant space-time, in general.

In the second section one presents specific properties of the Euclidean world function, which form the necessary and sufficient conditions of the Euclideaness. The third and fourth sections are devoted to consideration of the timelike vectors equivalence. In the fifth section the equivalence of null vectors is considered. Construction of geometrical objects is considered in the sixth section. Temporal evolution of the timelike straight line segment is considered in the seventh section.

2 Specific properties of the n-dimensional proper Euclidean space

There are four conditions which are necessary and sufficient conditions of the fact, that the world function σ is the world function of n-dimensional Euclidean space [4]. They have the form:

I. Definition of the dimension:

$$\exists\mathcal{P}^n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \qquad F_n(\mathcal{P}^n) \neq 0, \qquad F_k\left(\Omega^{k+1}\right) = 0, \qquad k > n \tag{2.1}$$

where $F_n(\mathcal{P}^n)$ is the Gram's determinant (1.8). Vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$ are basic vectors of the rectilinear coordinate system K_n with the origin at the point P_0. The metric tensors $g_{ik}(\mathcal{P}^n)$, $g^{ik}(\mathcal{P}^n)$, $i, k = 1, 2, ...n$ in K_n are defined by the relations

$$\sum_{k=1}^{k=n} g^{ik}(\mathcal{P}^n)\, g_{lk}(\mathcal{P}^n) = \delta_l^i, \qquad g_{il}(\mathcal{P}^n) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_l), \qquad i, l = 1, 2, ...n \tag{2.2}$$

$$F_n(\mathcal{P}^n) = \det||g_{ik}(\mathcal{P}^n)|| \neq 0, \qquad i, k = 1, 2, ...n \tag{2.3}$$

II. Linear structure of the Euclidean space:

$$\sigma(P, Q) = \frac{1}{2}\sum_{i,k=1}^{i,k=n} g^{ik}(\mathcal{P}^n)\,(x_i(P) - x_i(Q))\,(x_k(P) - x_k(Q)), \qquad \forall P, Q \in \Omega \tag{2.4}$$

where coordinates $x_i(P), x_i(Q), i = 1, 2, ...n$ of the points P and Q are covariant coordinates of the vectors $\mathbf{P}_0\mathbf{P}$, $\mathbf{P}_0\mathbf{Q}$ respectively, defined by the relation

$$x_i(P) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}), \qquad i = 1, 2, ...n \tag{2.5}$$

III: The metric tensor matrix $g_{lk}(\mathcal{P}^n)$ has only positive eigenvalues

$$g_k > 0, \qquad k = 1, 2, ..., n \tag{2.6}$$

IV. The continuity condition: the system of equations

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}) = y_i \in \mathbb{R}, \qquad i = 1, 2, ...n \tag{2.7}$$

considered to be equations for determination of the point P as a function of coordinates $y = \{y_i\}$, $i = 1, 2, ...n$ has always one and only one solution. Conditions I – IV contain a reference to the dimension n of the Euclidean space.

3 Equivalence of two vectors

The property of the two vectors equality may be introduced in any T-geometry by means of the relation (1.14). But in the arbitrary T-geometry the equality of two vectors is intransitive, in general, because of the parallelism multivariance. Intransitivity and multivariance of the two vectors equality is very inconvenient in applications. We shall use the term "equivalence" instead of the term "equality".

Definition. Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent ($\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1$), if the conditions (1.14) take place

$$\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1: \qquad ((\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1|\cdot|\mathbf{Q}_0\mathbf{Q}_1|)\wedge(|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|) \tag{3.1}$$

It follows from (3.1), that if $(\mathbf{P}_0\mathbf{P}_1)\text{eqv}(\mathbf{Q}_0\mathbf{Q}_1)$, then $(\mathbf{Q}_0\mathbf{Q}_1)\text{eqv}(\mathbf{P}_0\mathbf{P}_1)$

Remark. We distinguish between the equality (=) of vectors and equivalence (eqv) of vectors. For instance, the equality $\mathbf{P}_0\mathbf{P}_1 = \mathbf{P}_0\mathbf{Q}_1$ means, that the points P_1 and Q_1 coincide ($P_1 = Q_1$). Equality $\mathbf{P}_0\mathbf{P}_1 = \mathbf{Q}_0\mathbf{Q}_1$ means, that the point P_1 coincides with Q_1 ($P_1 = Q_1$) and the point P_0 coincides with Q_0 ($P_0 = Q_0$), whereas equivalence $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1$ means the fulfilment of relations (3.1). The point P_0 may not coincide with Q_0 and the point P_1 may not coincide with Q_1, i.e. equalities $P_0 = Q_0$ and $P_1 = Q_1$ may not take place.

The shift vector $\mathbf{P}_0\mathbf{Q}_0$ describes the shift of the origin P_0 of the vector $\mathbf{P}_0\mathbf{P}_1$. The shift vector $\mathbf{P}_1\mathbf{Q}_1$ describes the shift of the end P_1 of the vector $\mathbf{P}_0\mathbf{P}_1$. In the proper Euclidean space equivalence of shift vectors $\mathbf{P}_0\mathbf{Q}_0$ and $\mathbf{P}_1\mathbf{Q}_1$ leads to equivalence of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ and vice versa equivalence of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ leads to equivalence of their shift vectors $\mathbf{P}_0\mathbf{Q}_0\text{eqv}\mathbf{P}_1\mathbf{Q}_1$. In the general T-geometry the equivalence of shift vectors $\mathbf{P}_0\mathbf{Q}_0$ and $\mathbf{P}_1\mathbf{Q}_1$ is not sufficient for equivalence of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$. It is necessary once more constraint $|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|$ or $\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1$, to provide their equivalence.

Theorem. Vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent, if shift vectors $\mathbf{P}_0\mathbf{Q}_0$ and $\mathbf{P}_1\mathbf{Q}_1$ are equivalent and $|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|$, or $\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1$

Let ($\mathbf{P}_0\mathbf{Q}_0\text{eqv}\mathbf{P}_1\mathbf{Q}_1$). Equivalence of $\mathbf{P}_0\mathbf{Q}_0$ and $\mathbf{P}_1\mathbf{Q}_1$ is written in the form of two relations

$$\sigma(P_0,Q_1) + \sigma(P_1,Q_0) - \sigma(P_0,P_1) - \sigma(Q_0,Q_1) = 2\sqrt{\sigma(P_0,Q_0)\,\sigma(P_1,Q_1)} \tag{3.2}$$

$$\sigma(P_0,Q_0) = \sigma(P_1,Q_1) \tag{3.3}$$

In force of (3.3) equation (3.2) may be written in the form

$$\sigma(P_0,Q_1) + \sigma(P_1,Q_0) - \sigma(P_0,P_1) - \sigma(Q_0,Q_1) = \sigma(P_0,Q_0) + \sigma(P_1,Q_1) \tag{3.4}$$

The relation $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1$ is written in the form

$$\sigma(P_0,Q_1) + \sigma(P_1,Q_0) - \sigma(P_0,Q_0) - \sigma(P_1,Q_1) = 2\sqrt{\sigma(P_0,P_1)\,\sigma(Q_0,Q_1)} \tag{3.5}$$

$$\sigma(P_0,P_1) - \sigma(Q_0,Q_1) = 0 \tag{3.6}$$

The difference of (3.4) and (3.5) has the form

$$\sigma(P_0,P_1)+\sigma(Q_0,Q_1)=2\sqrt{\sigma(P_0,P_1)\,\sigma(Q_0,Q_1)} \tag{3.7}$$

which can be reduced to the form

$$\left(\sqrt{\sigma(P_0,P_1)}-\sqrt{\sigma(Q_0,Q_1)}\right)^2=0 \tag{3.8}$$

Let now $|\mathbf{P}_0\mathbf{P}_1|=|\mathbf{Q}_0\mathbf{Q}_1|$, and relations (3.6), (3.7) take place. As far as the relations (3.2), (3.3) are supposed to be fulfilled, the relation (3.4) is fulfilled also. The relation (3.5) takes place also, because it is a sum of relations (3.4) and (3.7). Thus, equations (3.5) and (3.6) are fulfilled. It means that $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1$.

Let now $|\mathbf{P}_0\mathbf{P}_1|\upuparrows|\mathbf{Q}_0\mathbf{Q}_1|$, and relation (3.5) is fulfilled. As far as the relations (3.2), (3.3), (3.4) are supposed to be fulfilled, the relation (3.7) takes place also, because the relation (3.7) is a difference of equations (3.4) and (3.5). Equation (3.6) is a corollary of (3.7). Thus, equations (3.5), (3.6) are fulfilled and $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1$. The theorem is proved.

Note, that in the proper Euclidean space, where the concept of equivalence is single-variant and transitive, the equivalence may be replaced by the equality, and the relations $\mathbf{P}_0\mathbf{Q}_0\text{eqv}\mathbf{P}_1\mathbf{Q}_1$ and $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1$ may be written respectively in the form $\mathbf{P}_0\mathbf{Q}_0=\mathbf{P}_1\mathbf{Q}_1$ and $\mathbf{P}_0\mathbf{P}_1=\mathbf{Q}_0\mathbf{Q}_1$. (Here symbol " = " means single-variant equivalence. It is used in the sense, which it has in the Euclidean geometry.) These relations are equivalent, and the additional condition $|\mathbf{P}_0\mathbf{P}_1|=|\mathbf{Q}_0\mathbf{Q}_1|$ is a corollary of any of these relations. Indeed, adding vector $\mathbf{P}_1\mathbf{Q}_0$ to both sides of the equality

$$\mathbf{P}_0\mathbf{P}_1=\mathbf{Q}_0\mathbf{Q}_1 \tag{3.9}$$

we obtain

$$\mathbf{P}_0\mathbf{Q}_0=\mathbf{P}_1\mathbf{Q}_1 \tag{3.10}$$

Besides, in the proper Euclidean geometry the relation $\mathbf{P}_1\mathbf{P}_2\text{eqv}\mathbf{Q}_1\mathbf{Q}_2$ is a corollary of the relations $(\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1)\wedge(\mathbf{P}_0\mathbf{P}_2\text{eqv}\mathbf{Q}_0\mathbf{Q}_2)$. To prove this, we write these relations in the form of equalities

$$\mathbf{P}_0\mathbf{P}_1=\mathbf{Q}_0\mathbf{Q}_1,\qquad \mathbf{P}_0\mathbf{P}_2=\mathbf{Q}_0\mathbf{Q}_2 \tag{3.11}$$

Subtracting the first relation (3.11) from the second one, we obtain

$$\mathbf{P}_1\mathbf{P}_2=\mathbf{Q}_1\mathbf{Q}_2 \tag{3.12}$$

Thus, in the proper Euclidean geometry among six relations $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{P}_0\mathbf{P}_2\text{eqv}\mathbf{Q}_0\mathbf{Q}_2$, $\mathbf{P}_1\mathbf{P}_2\text{eqv}\mathbf{Q}_1\mathbf{Q}_2$ there are only four independent conditions, whereas in the general case all six conditions are independent, in general.

4 Examples of equivalent vectors in uniform isotropic multivariant space-time geometry

We consider the σ-space $V_{\rm d}=\{\sigma,\mathbb{R}^4\}$ with the world function

$$\sigma_{\rm d}=\left\{\begin{array}{cc}\sigma_{\rm M}+d, & \text{if}\ \ \sigma_{\rm M}>0\\ \sigma_{\rm M}, & \text{if}\ \ \sigma_{\rm M}\leq 0\end{array}\right., \qquad d=\lambda_0^2=\text{const}>0 \tag{4.1}$$

where $\sigma_{\rm M}$ is the world function of the 4-dimensional space-time of Minkowski. In the inertial coordinate system the world function $\sigma_{\rm M}$ has the form

$$\sigma_{\rm M}\left(P,P'\right)=\sigma_{\rm M}\left(x,x'\right)=\left(x^0-x'^0\right)^2-\left(\mathbf{x}-\mathbf{x}'\right)^2 \tag{4.2}$$

where coordinates of points P and P' are $P=\{x^0,\mathbf{x}\}=\{ct,x^1,x^2,x^3\}$, $P'=\{x'^0,\mathbf{x}'\}=\{ct',x'^1,x'^2,x'^3\}$ and c is the speed of the light. The constant d is qualified as a distortion of the distorted space-time $V_{\rm d}$, described by the world function σ. The constant λ_0 may be considered as an "elementary length" associated with the distorted space-time $V_{\rm d}$.

The space-time (4.1) is uniform and isotropic in the sense, that the world function $\sigma_{\rm d}$ is invariant with respect to the simultaneous Poincaré transformation of both arguments x and x'.

The continual space-time $V_{\rm d}$ demonstrates evidence of a discreteness in the sense, that there are no points x, x', separated by the timelike interval $\rho=\sqrt{2\sigma\left(x,x'\right)}$, with $\rho\in(0,\lambda_0)$. It seems rather unexpected, that the continual space-time may be simultaneously discrete. Apparently, discreteness of such a kind should be qualified as discreteness of time.

Let us consider two equivalent timelike vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{Q}_0\mathbf{Q}_1$. The points P_0,P_1, Q_0,Q_1 have coordinates

$$P_0=\{0,0,0,0\}, \qquad P_1=\{s,0,0,0\} \tag{4.3}$$

$$Q_0=\{a,b,0,0\}, \qquad Q_1=\{s+a+\alpha_0,b+\gamma_1,\gamma_2,\gamma_3\} \tag{4.4}$$

The time axis is chosen along the vector $\mathbf{P}_0\mathbf{P}_1$. The shift vector $\mathbf{P}_0\mathbf{Q}_0=\{a,b,0,0\}$ lies in the plane of coordinate axes x^0x^1. The length s of the vector $\mathbf{P}_0\mathbf{P}_1$ and parameters a,b of the shift are supposed to be given. The numbers $\alpha_0,\beta_1,\gamma_2,\gamma_3$ are to be determined from the condition that vectors

$$\mathbf{P}_0\mathbf{P}_1=\{s,0,0,0\}, \qquad \mathbf{Q}_0\mathbf{Q}_1=\{s+\alpha_0,\gamma_1,\gamma_2,\gamma_3\} \tag{4.5}$$

are equivalent.

As far the geometry is uniform and isotropic, the relations (4.3), (4.4) describe the general case of the points P_0,P_1,Q_0,Q_1 disposition with timelike vector $\mathbf{P}_0\mathbf{P}_1$. The specificity of formulas (4.3), (4.4) is obtained as a result of proper choice of the coordinate system.

We consider two different cases.

I. All points P_0, P_1, Q_0, Q_1 are different,

$$\sigma(P,Q) = \sigma_{\mathrm{M}}(P,Q) + \lambda_0^2, \qquad P \neq Q, \qquad P,Q \in \{P_0, P_1, Q_0, Q_1\} \tag{4.6}$$

II. $P_1 = Q_0$, all other points P_0, P_1, Q_1 are different, and all different points are separated by timelike intervals. In this case we have

$$\sigma(P,Q) = \sigma_{\mathrm{M}}(P,Q) + \lambda_0^2, \qquad P \neq Q, \qquad P,Q \in \{P_0, P_1, Q_1\} \tag{4.7}$$

$$\sigma(P_1,Q_0) = \sigma_{\mathrm{M}}(P_1,Q_0) = 0 \tag{4.8}$$

In the first case

$$|\mathbf{P}_0\mathbf{P}_1|^2 = |\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2 + 2\lambda_0^2, \qquad |\mathbf{Q}_0\mathbf{Q}_1|^2 = |\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{M}}^2 + 2\lambda_0^2 \tag{4.9}$$

$$|\mathbf{P}_0\mathbf{Q}_0|^2 = |\mathbf{P}_0\mathbf{Q}_0|_{\mathrm{M}}^2 + 2\lambda_0^2, \qquad |\mathbf{P}_1\mathbf{Q}_1|^2 = |\mathbf{P}_1\mathbf{Q}_1|_{\mathrm{M}}^2 + 2\lambda_0^2 \tag{4.10}$$

Here and in what follows the index "M" means that the quantity is calculated in the Minkowski space-time. Taking into account definition of the scalar product (1.6) and relations (4.6), we obtain

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{M}}, \qquad (\mathbf{P}_0\mathbf{Q}_0.\mathbf{P}_1\mathbf{Q}_1) = (\mathbf{P}_0\mathbf{Q}_0.\mathbf{P}_1\mathbf{Q}_1)_{\mathrm{M}} \tag{4.11}$$

Condition $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1$ in terms of the Minkowski quantities have the form

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{M}} = \sqrt{\left(|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2 + 2\lambda_0^2\right)\left(|\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{M}}^2 + 2\lambda_0^2\right)} \tag{4.12}$$

$$|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2 = |\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{M}}^2 \tag{4.13}$$

Using for $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{M}}$ and $|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2$, conventional expression in terms of coordinates, we obtain instead of (4.12) and (4.13) by means of (4.5)

$$s(s+\alpha_0) = s^2 + 2\lambda_0^2 \tag{4.14}$$

$$s^2 = (s+\alpha_0)^2 - \gamma_1^2 - \gamma_2^2 - \gamma_3^2 \tag{4.15}$$

Solution of these equations gives

$$\alpha_0 = \frac{2\lambda_0^2}{s}, \qquad \gamma_k = 2\lambda_0\sqrt{1+\frac{\lambda_0^2}{s^2}}\frac{q_k}{q}, \qquad k = 1,2,3 \tag{4.16}$$

where q_1, q_2, q_3 are arbitrary real constants and

$$q = \sqrt{q_1^2 + q_2^2 + q_3^2} \tag{4.17}$$

Thus, coordinates of the point Q_1

$$Q_1 = \left\{ s + a + \frac{2\lambda_0^2}{s}, b + 2\lambda_0\sqrt{1+\frac{\lambda_0^2}{s^2}}\frac{q_1}{q}, 2\lambda_0\sqrt{1+\frac{\lambda_0^2}{s^2}}\frac{q_2}{q}, 2\lambda_0\sqrt{1+\frac{\lambda_0^2}{s^2}}\frac{q_3}{q} \right\} \tag{4.18}$$

In the case, when $\lambda_0 \ll |s|$

$$Q_1 \approx \left\{ s + a + \frac{2\lambda_0^2}{s}, b + \frac{2\lambda_0 q_1}{q}, \frac{2\lambda_0 q_2}{q}, \frac{2\lambda_0 q_3}{q} \right\} \tag{4.19}$$

The correction, conditioned by distortion, to the coordinate x^0 is of the order λ_0^2, whereas the correction to other coordinates is of the order λ_0. Thus, the vector $\mathbf{Q}_0\mathbf{Q}_1$ is multivariant, and its multivariance is described by two arbitrary parameters.

The second case is more interesting from physical viewpoint, because it may be considered as a description of the temporal evolution of the geometrical object, described by two characteristic points P_0 and P_1, separated by the timelike interval. In the second case, when $P_1 = Q_0$, we have $a = s$, $b = 0$, i.e.

$$P_0 = \{0, 0, 0, 0\}, \qquad Q_0 = P_1 = \{s, 0, 0, 0\}, \qquad Q_1 = \{2s + \alpha_0, \gamma_1, \gamma_2, \gamma_3\} \tag{4.20}$$

The vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ have the same form (4.5), however in this case the relation between $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ and $(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{Q}_1)_{\mathrm{E}}$ has the form

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{Q}_1) = (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{Q}_1)_{\mathrm{M}} - \lambda_0^2, \tag{4.21}$$

which distinguishes from the relation (4.11).

Condition $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1$ in terms of Minkowskian quantities have the form

$$|\mathbf{P}_0\mathbf{Q}_0|^2 = |\mathbf{P}_0\mathbf{P}_1|^2 = |\mathbf{P}_0\mathbf{P}_1|^2_{\mathrm{M}} + 2\lambda_0^2, \qquad |\mathbf{P}_1\mathbf{Q}_0|^2 = |\mathbf{P}_1\mathbf{Q}_0|^2_{\mathrm{M}} = 0 \tag{4.22}$$

$$|\mathbf{P}_1\mathbf{Q}_1|^2 = |\mathbf{Q}_0\mathbf{Q}_1|^2 = |\mathbf{Q}_0\mathbf{Q}_1|^2_{\mathrm{M}} + 2\lambda_0^2 \tag{4.23}$$

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{M}} - \lambda_0^2 = \sqrt{\left(|\mathbf{P}_0\mathbf{P}_1|^2_{\mathrm{M}} + 2\lambda_0^2\right)\left(|\mathbf{Q}_0\mathbf{Q}_1|^2_{\mathrm{M}} + 2\lambda_0^2\right)} \tag{4.24}$$

$$|\mathbf{P}_0\mathbf{P}_1|^2_{\mathrm{M}} = |\mathbf{Q}_0\mathbf{Q}_1|^2_{\mathrm{M}} \tag{4.25}$$

They take the form

$$s(s + \alpha_0) - \lambda_0^2 = s^2 + 2\lambda_0^2 \tag{4.26}$$

$$s^2 = (s + \alpha_0)^2 - \gamma_1^2 - \gamma_2^2 - \gamma_3^2 \tag{4.27}$$

and distinguish from the relations (4.14), (4.15) by replacement of λ_0 by $\sqrt{3/2}\lambda_0$.

Using the change $a \to s$, $b \to 0$, $\lambda_0 \to \sqrt{3/2}\lambda_0$ in relations (4.18), (4.19), we obtain for the vector $\mathbf{Q}_0\mathbf{Q}_1$

$$\mathbf{Q}_0\mathbf{Q}_1 = \mathbf{P}_1\mathbf{Q}_1 = \left\{ s + \frac{3\lambda_0^2}{s}, \lambda_0\kappa\frac{q_1}{q}, \lambda_0\kappa\frac{q_2}{q}, \lambda_0\kappa\frac{q_3}{q} \right\} \tag{4.28}$$

where q_1, q_2, q_3 are arbitrary quantities

$$\kappa = \sqrt{6\left(1 + \frac{3\lambda_0^2}{2s^2}\right)} \tag{4.29}$$

and q is determined by the relation (4.17).

In the case, when $\lambda_0 \ll |s|$

$$\mathbf{Q}_0\mathbf{Q}_1 = \mathbf{P}_1\mathbf{Q}_1 \approx \left\{ s + \frac{3\lambda_0^2}{s}, \frac{\sqrt{6}\lambda_0 q_1}{q}, \frac{\sqrt{6}\lambda_0 q_2}{q}, \frac{\sqrt{6}\lambda_0 q_3}{q} \right\} \tag{4.30}$$

Another limit case, when $s \ll \lambda_0$. We set $s = \beta\lambda_0$, $\beta \ll 1$ and obtain

$$\mathbf{Q}_0\mathbf{Q}_1 = \mathbf{P}_1\mathbf{Q}_1 = \left\{ \lambda_0 \left(\beta + \frac{3}{\beta} \right), \lambda_0\kappa_1 \frac{q_1}{q}, \lambda_0\kappa_1 \frac{q_2}{q}, \lambda\kappa_1 \frac{q_3}{q} \right\} \tag{4.31}$$

where

$$\kappa_1 = \sqrt{6\left(1 + \frac{3}{2\beta^2}\right)} \tag{4.32}$$

In this case all coefficients $\alpha_0, \gamma_1, \gamma_2, \gamma_3$ are of the same order $3\lambda_0/\beta$, and multivariance of the vector $\mathbf{Q}_0\mathbf{Q}_1 = \mathbf{P}_1\mathbf{Q}_1$ is very large. We have for the $|\mathbf{Q}_0\mathbf{Q}_1|^2$

$$|\mathbf{Q}_0\mathbf{Q}_1|^2 = |\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{M}}^2 + 2\lambda_0^2 = \left(2 + \beta^2\right)\lambda_0^2 \tag{4.33}$$

5 Equivalence of two null vectors

Let us consider two equivalent null vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ in the space-time (4.1) $|\mathbf{P}_0\mathbf{P}_1| = 0$, $|\mathbf{Q}_0\mathbf{Q}_1| = 0$. We consider the case, when the points P_1 and Q_0 coincide $P_1 = Q_0$. Then

$$P_0 = \{0, 0, 0, 0\}, \qquad P_1 = \{s, s, 0, 0\}, \tag{5.1}$$

$$Q_0 = \{s, s, 0, 0\}, \qquad Q_1 = \{2s + \alpha_0, 2s + \gamma_1, \gamma_2, \gamma_3\}, \tag{5.2a}$$

$$\mathbf{P}_0\mathbf{P}_1 = \{s, s, 0, 0\}, \qquad \mathbf{P}_1\mathbf{Q}_1 = \mathbf{Q}_0\mathbf{Q}_1 = \{s + \alpha_0, s + \gamma_1, \gamma_2, \gamma_3\} \tag{5.3}$$

$$\mathbf{P}_0\mathbf{Q}_1 = \{2s + \alpha_0, 2s + \gamma_1, \gamma_2, \gamma_3\} \tag{5.4}$$

In this case we obtain

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) = \sigma(P_0, Q_1) \tag{5.5}$$

The equivalence conditions have the form

$$|\mathbf{P}_0\mathbf{P}_1|^2 = |\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2 = 0, \qquad |\mathbf{P}_1\mathbf{Q}_2|^2 = |\mathbf{P}_1\mathbf{Q}_2|_{\mathrm{M}}^2 = 0 \tag{5.6}$$

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{Q}_2) = \sigma(P_0, Q_2) - \sigma(P_1, Q_2) - \sigma(P_0, P_1) = \sigma(P_0, Q_2) = 0 \tag{5.7}$$

In terms of coordinates we have

$$\sigma(P_0, Q_2) = \sigma_{\mathrm{M}}(P_0, Q_2) = (2s + \alpha_0)^2 - (2s + \gamma_1)^2 - \gamma_2^2 - \gamma_3^2 = 0 \tag{5.8}$$

$$(s + \alpha_0)^2 - (s + \gamma_1)^2 - \gamma_2^2 - \gamma_3^3 = 0 \tag{5.9}$$

Solution of equations (5.8), (5.9) has the form

$$\alpha_0 = \gamma_1, \qquad \gamma_2 = \gamma_3 = 0$$

$$\mathbf{P}_1\mathbf{Q}_2 = \{s + \alpha_0, s + \alpha_0, 0, 0\}, \qquad \mathbf{P}_0\mathbf{Q}_2 = \{2s + \alpha_0, 2s + \alpha_0, 0, 0\} \tag{5.10}$$

where α_0 is an arbitrary real quantity.

6 Construction of geometrical objects in T-geometry

Geometrical object $\mathcal{O} \subset \Omega$ is a subset of points in the point space Ω. In the T-geometry the geometric object $\mathcal{O}$ is described by means of the skeleton-envelope method [4]. It means that any geometric object $\mathcal{O}$ is considered to be a set of intersections and joins of elementary geometric objects (EGO).

The finite set $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \subset \Omega$ of parameters of the envelope function $f_{\mathcal{P}^n}$ is the skeleton of elementary geometric object (EGO) $\mathcal{E} \subset \Omega$. The set $\mathcal{E} \subset \Omega$ of points forming EGO is called the envelope of its skeleton $\mathcal{P}^n$. In the continuous generalized geometry the envelope $\mathcal{E}$ is usually a continual set of points. The envelope function $f_{\mathcal{P}^n}$

$$f_{\mathcal{P}^n}: \qquad \Omega \rightarrow \mathbb{R}, \tag{6.1}$$

determining EGO, is a function of the running point $R \in \Omega$ and of parameters $\mathcal{P}^n \subset \Omega$. The envelope function $f_{\mathcal{P}^n}$ is supposed to be an algebraic function of s arguments $w = \{w_1, w_2, ...w_s\}$, $s = (n+2)(n+1)/2$. Each of arguments $w_k = \sigma(Q_k, L_k)$ is the world function σ of two arguments $Q_k, L_k \in \{R, \mathcal{P}^n\}$, either belonging to skeleton $\mathcal{P}^n$, or coinciding with the running point R. Thus, any elementary geometric object $\mathcal{E}$ is determined by its skeleton $\mathcal{P}^n$ and its envelope function $f_{\mathcal{P}^n}$ as the set of zeros of the envelope function

$$\mathcal{E} = \{R | f_{\mathcal{P}^n}(R) = 0\} \tag{6.2}$$

Characteristic points of the EGO are the skeleton points $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\}$. The simplest example of EGO is the segment $\mathcal{T}_{[P_0P_1]}$ of the straight line (1.24) between the points P_0 and P_1, which is defined by the relation

$$\begin{aligned} \mathcal{T}_{[P_0P_1]} &= \{R | f_{P_0P_1}(R) = 0\}, \\ f_{P_0P_1}(R) &= \sqrt{2\sigma(P_0, R)} + \sqrt{2\sigma(R, P_1)} - \sqrt{2\sigma(P_0, P_1)} \end{aligned} \tag{6.3}$$

Another example is the sphere $\mathcal{S}_{OQ}$, where O is the center of the sphere and Q is some point on the surface of the sphere. The sphere $\mathcal{S}_{P_0P_1}$ is described by the relation

$$\mathcal{S}_{OQ} = \{R | g_{OQ}(R) = 0\}, \qquad g_{OQ}(R) = \sqrt{2\sigma(O, R)} - \sqrt{2\sigma(O, Q)} \tag{6.4}$$

Here points O, Q form the skeleton of the sphere, whereas the function g_{OQ} is the envelope function.

The third example is the cylinder $\mathcal{C}(P_0, P_1, Q)$ with the points P_0, P_1 on the cylinder axis and the point Q on its surface. The cylinder $\mathcal{C}(P_0, P_1, Q)$ is determined by the relation

$$\begin{aligned} \mathcal{C}(P_0, P_1, Q) &= \{R | f_{P_0P_1Q}(R) = 0\}, \\ f_{P_0P_1Q}(R) &= F_2(P_0, P_1, Q) - F_2(P_0, P_1, R) \end{aligned} \tag{6.5}$$

$$F_2(P_0, P_1, Q) = \begin{vmatrix} (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_1) & (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}) \\ (\mathbf{P}_0\mathbf{Q}.\mathbf{P}_0\mathbf{P}_1) & (\mathbf{P}_0\mathbf{Q}.\mathbf{P}_0\mathbf{Q}) \end{vmatrix} \tag{6.6}$$

Here $\sqrt{F_2(P_0,P_1,Q)}$ is the area of the parallelogram, constructed on the vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{Q}$ and $\frac{1}{2}\sqrt{F_2(P_0,P_1,Q)}$ is the area of triangle with vertices at the points P_0,P_1,Q. The equality $F_2(P_0,P_1,Q)=F_2(P_0,P_1,R)$ means that the distance between the point Q and the axis, determined by the vector $\mathbf{P}_0\mathbf{P}_1$, is equal to the distance between R and the axis. Here the points P_0,P_1,Q form the skeleton of the cylinder, whereas the function $f_{P_0P_1Q}$ is the envelope function.

Definition. Two EGOs $\mathcal{E}(\mathcal{P}^n)$ and $\mathcal{E}(\mathcal{Q}^n)$ are equivalent, if their skeletons are equivalent and their envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ are equal. Equivalence of two skeletons $\mathcal{P}^n\equiv\{P_0,P_1,...,P_n\}\subset\Omega$ and $\mathcal{Q}^n\equiv\{Q_0,Q_1,...,Q_n\}\subset\Omega$ means that

$$\mathbf{P}_i\mathbf{P}_k\text{eqv}\mathbf{Q}_i\mathbf{Q}_k,\qquad i,k=0,1,...n,\quad i<k \tag{6.7}$$

Equivalence of the envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ means that

$$f_{\mathcal{P}^n}(R)=\Phi\left(g_{\mathcal{P}^n}(R)\right),\qquad \forall R\in\Omega \tag{6.8}$$

where Φ is an arbitrary function, having the property

$$\Phi:\mathbb{R}\to\mathbb{R},\qquad \Phi(0)=0 \tag{6.9}$$

Equivalence of shapes of two EGOs $\mathcal{E}(\mathcal{P}^n)$ and $\mathcal{E}(\mathcal{Q}^n)$ is determined by equivalence of shapes of their skeletons $\mathcal{P}^n$ and $\mathcal{Q}^n$, which is described by the relations

$$|\mathbf{P}_i\mathbf{P}_k|=|\mathbf{Q}_i\mathbf{Q}_k|,\qquad i,k=0,1,...n,\quad i<k \tag{6.10}$$

and equivalence of their envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ (6.8).

Equivalence of orientations of skeletons $\mathcal{P}^n$ and $\mathcal{Q}^n$ in the point space Ω is described by the relations

$$\mathbf{P}_i\mathbf{P}_k\upuparrows\mathbf{Q}_i\mathbf{Q}_k,\qquad i,k=0,1,...n,\quad i<k \tag{6.11}$$

Equivalence of shapes and orientations of skeletons is equivalence of skeletons, described by the relations (6.7).

Definition. The elementary geometric object $\mathcal{E}(\mathcal{P}^n)$ exists, if at any time moment there is an elementary geometric object $\mathcal{E}'(\mathcal{P}'^n)$, which is equivalent to EGO $\mathcal{E}(\mathcal{P}^n)$. We suppose, the skeleton $\mathcal{P}^n=\{P_0,P_1,...P_n\}$ contains points separated by the timelike interval. We suppose that the vector $\mathbf{P}_0\mathbf{P}_1$ is timelike ($|\mathbf{P}_0\mathbf{P}_1|^2>0$). We assume, that the elementary geometrical object $\mathcal{E}(\mathcal{P}^n)=\mathcal{E}(P_0,P_1,...,P_n)$ is placed at the point P_0. The same EGO placed at the point P_1 has the form $\mathcal{E}(P_0',P_1',P_2'...,P_n')$ with $P_0'=P_1$. The points P_0 and $P_0'=P_1$ are separated by the timelike interval, and we may consider the EGO $\mathcal{E}(P_0',P_1',P_2'...,P_n')$ as a result of temporal evolution of the EGO $\mathcal{E}(P_0,P_1,...,P_n)$, provided these objects are equivalent, i.e.

$$\mathbf{P}_i\mathbf{P}_k\text{eqv}\mathbf{P}_i'\mathbf{P}_k',\qquad i,k=0,1,...n,\quad i<k \tag{6.12}$$

Thus, if EGOs $\mathcal{E}\left(P_0^{(0)},P_1^{(0)},...,P_n^{(0)}\right)$, $\mathcal{E}\left(P_0^{(1)},P_1^{(1)},...,P_n^{(1)}\right)$, $\mathcal{E}\left(P_0^{(2)},P_1^{(2)},...,P_n^{(2)}\right)$,... $\mathcal{E}\left(P_0^{(k)},P_1^{(k)},...,P_n^{(k)}\right)$,... are equivalent in pairs

$$\mathcal{E}\left(P_0^{(k-1)},P_1^{(k-1)},...,P_n^{(k-1)}\right)\text{eqv}\mathcal{E}\left(P_0^{(k)},P_1^{(k)},...,P_n^{(k)}\right),\qquad k=1,2,... \tag{6.13}$$

and

$$
\begin{aligned}
P_1^{(0)} &= P_0^{(1)}, \quad P_1^{(1)} = P_0^{(2)}, \quad P_1^{(2)} = P_0^{(3)}, ...P_1^{(k)} = P_0^{(k+1)}, ... &(6.14)\\
\left|\mathbf{P}_0^{(k)}\mathbf{P}_1^{(k)}\right|^2 &> 0, \qquad k = 1, 2, ... &(6.15)
\end{aligned}
$$

one may consider existence of the set of elementary geometrical objects (EGOs) $\mathcal{E}\left(P_0^{(0)}, P_1^{(0)}, ..., P_n^{(0)}\right)$, $\mathcal{E}\left(P_0^{(1)}, P_1^{(1)}, ..., P_n^{(1)}\right)$, $\mathcal{E}\left(P_0^{(2)}, P_1^{(2)}, ..., P_n^{(2)}\right)$,... $\mathcal{E}\left(P_0^{(k)}, P_1^{(k)}, ..., P_n^{(k)}\right)$, with properties (6.13) - (6.15) as a temporal evolution of EGO $\mathcal{E}\left(P_0^{(0)}, P_1^{(0)}, ..., P_n^{(0)}\right)$.

Thus, the space-time geometry determines a possibility of existence of the geometric object $\mathcal{E}(\mathcal{P}^n)$ and its temporal evolution. Some objects may exist, other ones do not exist. This fact depends on possibility of fulfilment of the relation (6.13). For some geometrical objects the temporal evolution may be multivariant, for other ones it is single-variant. It is possible such geometrical objects, for which there is no equivalent geometrical objects, and there is no temporal evolution.

7 Temporal evolution of timelike segment of the straight

Timelike segment (6.3) of the straight line (1.24) is an elementary geometrical object $\mathcal{T}_{[P_0P_1]}$, described by the skeleton, consisting of two points P_0, P_1. Temporal evolution of this segment is described by the broken tube $\mathcal{T}_{\text{br}}$.

$$
\mathcal{T}_{\text{br}} = \bigcup_i \mathcal{T}_{[P_iP_{i+1}]}, \qquad \mathbf{P}_{i-1}\mathbf{P}_i\text{eqv}\mathbf{P}_i\mathbf{P}_{i+1}, \qquad \left|\mathbf{P}_i\mathbf{P}_{i+1}\right|^2 = \mu^2, \qquad i = 0, \pm 1, \pm 2, ... \tag{7.1}
$$

The shape of $\mathcal{T}_{\text{br}}$ in the space-time (4.1) is multivariant. It looks as a chain, consisting of similar links $\mathcal{T}_{[P_iP_{i+1}]}$. In the Minkowski space-time the shape of $\mathcal{T}_{\text{br}}$ is single-variant, and the chain of links $\mathcal{T}_{[P_iP_{i+1}]}$ degenerates into the timelike straight line.

In the inertial coordinate system $\{ct, x^1, x^2, x^3\}$, where the points P_0, P_1 have coordinates

$$
P_0 = \{0, 0, 0, 0\}, \qquad P_1 = \left\{\sqrt{\mu^2 - 2\lambda_0^2}, 0, 0, 0\right\}, \tag{7.2}
$$

and the vector $\mathbf{P}_0\mathbf{P}_1$ has the length

$$
|\mathbf{P}_0\mathbf{P}_1| = \sqrt{|\mathbf{P}_0\mathbf{P}_1|_{\text{M}}^2 + 2\lambda_0^2} = \mu, \tag{7.3}
$$

the surface $\mathcal{T}_{(P_0P_1)} = \mathcal{T}_{[P_0P_1]} \backslash \{P_0, P_1\}$ is described by the following equation

$$
r^2 = \mathbf{x}^2 = \frac{2\lambda_0^2\left(ct - \frac{1}{2}\sqrt{\mu^2 - 2\lambda_0^2}\right)^2}{\mu^2} + \frac{3}{2}\lambda_0^2, \qquad 0 < ct < \sqrt{\mu^2 - 2\lambda_0^2} \tag{7.4}
$$

The skeleton points P_0, P_1 do not belong to the surface (7.4), but the belong to $\mathcal{T}_{[P_0P_1]}$. This surface is a tube with minimal radius $r_{\min} = \sqrt{3/2}\lambda_0$ and maximal radius $r_{\max} = \sqrt{2\lambda_0^2 - \frac{\lambda_0^4}{\mu^2}}$, $\mu > \sqrt{2}\lambda_0$. In the limit $\lambda_0 \to 0$ the maximal and minimal radii tend to zero, and the tube (7.4) degenerates into the straight line interval.

Remark. The form of the envelope function is of no importance for the temporal evolution of the geometrical object, described by two skeleton points P_0, P_1, In particular, one may consider the sphere $\mathcal{S}_{P_0P_1}$, defined by the relation (6.4), instead of EGO $\mathcal{T}_{[P_0P_1]}$. In this case we have instead of the broken tube (7.1)

$$\mathcal{T}_{\rm br} = \bigcup_i \mathcal{S}_{P_iP_{i+1}}, \qquad \mathbf{P}_{i-1}\mathbf{P}_i \text{eqv} \mathbf{P}_i\mathbf{P}_{i+1}, \qquad |\mathbf{P}_i\mathbf{P}_{i+1}|^2 = \mu^2, \qquad i = 0, \pm1, \pm2, ... \tag{7.5}$$

Here instead of the tube segment (7.4), we have the hyperbola

$$r^2 = \mathbf{x}^2 = c^2t^2 - \mu^2 + 2\lambda_0^2 \tag{7.6}$$

Defect of the presentation (7.5) is determined by the fact that one of the skeleton points (P_0) does not belong to the sphere envelope $\mathcal{S}_{P_0P_1}$. In the limit $\lambda_0 \to 0$ the set of hyperbolas (7.6) is not associated with the particle world line.

The mutual location of two adjacent links $\mathcal{T}_{[P_0P_1]}$ and $\mathcal{T}_{[P_1P_2]}$ is described by the angle between the vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$. As far as these vectors are equivalent and, hence, are in parallel this angle $\theta = 0$, because

$$\cosh\theta = \frac{(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)}{|\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{P}_1\mathbf{P}_2|} = 1 \tag{7.7}$$

However, as far as

$$|\mathbf{P}_0\mathbf{P}_1|^2_{\rm M} = |\mathbf{P}_1\mathbf{P}_2|^2_{\rm M} = |\mathbf{P}_0\mathbf{P}_1|^2 - 2\lambda_0^2, \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\rm M} = (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2) - \lambda_0^2 \tag{7.8}$$

the angle $\theta_{\rm M}$ between vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$, measured on the Minkowski manifold is defined by the relation

$$\cosh\theta_{\rm M} = \frac{(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\rm M}}{|\mathbf{P}_0\mathbf{P}_1|_{\rm M} \cdot |\mathbf{P}_1\mathbf{P}_2|_{\rm M}} = \frac{(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2) - \lambda_0^2}{|\mathbf{P}_0\mathbf{P}_1|^2 - 2\lambda_0^2} = \frac{\mu^2 - \lambda_0^2}{\mu^2 - 2\lambda_0^2} > 1 \tag{7.9}$$

In the case, when $\mu \gg \lambda_0$, it follows from (7.9), that

$$\theta_{\rm M} = \sqrt{2}\frac{\lambda_0}{\mu} \tag{7.10}$$

Thus, at fixed vector $\mathbf{P}_0\mathbf{P}_1$ the point P_2, determining the adjacent vector $\mathbf{P}_1\mathbf{P}_2$, lies on the surface of the cone with angle $\theta_{\rm M}$ at the vertex P_1. Thus, if $\lambda_0 \neq 0$, the broken tube (7.1) is multivariant. If $\lambda_0 \to 0$, then according to (7.10) $\theta_{\rm M} \to 0$, and the cone degenerates into the straight line. The broken tube (7.1) degenerates into the straight line, which associates with the world line of a particle. In the

Minkowski space-time the world line is a geometric characteristic of a particle. However, besides, the particle has dynamic characteristics: the momentum $\mathbf{p}$ and the mass, which cannot be determined geometrically in the Minkowski space-time. The momentum vector is tangent to the world line, and its direction may be determined by the shape of the world line. However, its length (the particle mass) cannot be determined geometrically. In the Minkowski space-time the mass is a non-geometric characteristic, associated with the particle world line.

In the distorted space-time $V_{\rm d}$, described by the world function (4.1), the particle mass is defined geometrically as the length $|\mathbf{P}_i\mathbf{P}_{i+1}| = \mu$ of the link of the broken tube (7.1). Indeed, in $V_{\rm d}$ the link $\mathcal{T}_{[P_iP_{i+1}]}$ is a tube segment of the radius r. The ends P_i and P_{i+1} of this segment can be determined geometrically. If we know the ends P_i, P_{i+1} of $\mathcal{T}_{[P_iP_{i+1}]}$, we can determine the length $|\mathbf{P}_i\mathbf{P}_{i+1}| = \mu$. But in this case the mass μ is determined in the units of length as the distance between the skeleton points P_i, P_{i+1} of the segment $\mathcal{T}_{[P_iP_{i+1}]}$. The skeleton points P_0, P_1 do not belong to the surface (7.4), which describes the interval $\mathcal{T}_{(P_iP_{i+1})} = \mathcal{T}_{[P_iP_{i+1}]} \setminus \{P_0, P_1\}$. Usually the particle mass m is expressed in the units of mass (g), and there is an universal transferring coefficient b, connecting the usual mass m with the geometric mass μ

$$m = b\mu = b\,|\mathbf{P}_i\mathbf{P}_{i+1}|\,, \qquad [b] = \text{g/cm} \tag{7.11}$$

The same coefficient is used for connection of the geometric vector $\mathbf{P}_i\mathbf{P}_{i+1}$ with the physical momentum 4-vector p_k

$$p_k = bc\,(\mathbf{P}_i\mathbf{P}_{i+1})_k\,, \qquad k = 0,1,2,3 \tag{7.12}$$

where c is the speed of the light and $(\mathbf{P}_i\mathbf{P}_{i+1})_k$ are coordinates of the vector $\mathbf{P}_i\mathbf{P}_{i+1}$ in some inertial coordinate system.

Thus, conceptually the dynamics of a particle is a corollary of a geometrical description. However, the conventional dynamic formalism for description of the free particle motion is suited only for description of single-variant motion. In this sense the conventional single-variant dynamics agrees with the Minkowski space-time geometry, whereas it disagrees with the multivariant motion in the distorted space-time $V_{\rm d}$.

From the viewpoint of the distorted geometry $\mathcal{G}_{\rm d}$ the interval $\mathcal{T}_{(P_0P_1)}$ describes the particle, which has the shape of a hallow 3-sphere of the radius r ($r_{\min} < r < r_{\max}$) . Such a spherical particle exists at rest in the coordinate system K_0 during the proper time t ($0 < t < \mu/c$). At the proper time $t = \mu/c$ the spherical particle degenerates into the pointlike particle, located at the point $P_1 = \{\mu, 0, 0, 0\}$. At the next time moment $t > \mu/c$ the pointlike particle turns into spherical particle of the radius r, ($r_{\min} < r < r_{\max}$) and moves in the random spatial direction with the speed $|\mathbf{v}| = c{\cdot}\text{arth}(\theta_{\rm M}) = c{\cdot}\text{arth}\left(\sqrt{2}\frac{\lambda_0}{\mu}\right)$ in the coordinate system K_0. Simultaneously this spherical particle is at rest in some coordinate system K_1, moving with respect to the coordinate system K_0 with the velocity $\mathbf{v}$, ($|\mathbf{v}| = c \cdot \text{arth}\,(\theta_{\rm M})$). The spherical particle is at rest in the coordinate system K_1 during the proper time t, $\mu/c < t < 2\mu/c$. At the proper time $t = 2\mu/c$ the spherical particle degenerates into the pointlike particle at the point P_2 and so on. The classical mechanics cannot describe such a geometrical object as the spherical particle without description of internal degrees of freedom. It cannot also describe transformation of a spherical particle into a pointlike particle and vice versa.

There is a problem of construction of the multivariant dynamics, which would agree with the distorted geometry $\mathcal{G}_{\rm d}$, described by the world function (4.1). To describe multivariant motion of a particle, one needs to consider all variants of motion simultaneously and to obtain some average description (some average world lines of a particle). Such a description we shall produce on the Minkowski manifold, where results of all mathematical operations (equality, summation, multiplication, differentiation, etc.) are defined uniquely. Working on the Minkowski manifold and using single-valued operation, defined on this manifold, we shall use geometry and properties of geometrical objects, defined by the world function (4.1).

In general, the classical dynamics has some experience of multivariant motion description. In the case, when there are dynamic equations for a single dynamic system, but there are different variants of the initial conditions, the motion appears to be multivariant, because of different variants of initial conditions. In this case one uses the statistical ensemble as a dynamic system, consisting of many identical independent dynamic systems.

For instance, a free nonrelativistic particle is described by the action

$$\mathcal{A}[\mathbf{x}] = \int m \left(\frac{d\mathbf{x}}{dt}\right)^2 dt \tag{7.13}$$

where $\mathbf{x} = \mathbf{x}(t)$ describes the world line of the classical pointlike particle.

The statistical ensemble of free nonrelativistic particles is the dynamic system, described by the action

$$\mathcal{A}[\mathbf{x}] = \int m \left(\frac{d\mathbf{x}}{dt}\right)^2 dt d\boldsymbol{\xi} \tag{7.14}$$

where $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, $\boldsymbol{\xi} = \{\xi_1, \xi_2, ...\xi_n\}$. The independent variables $\boldsymbol{\xi}$ label particles of the statistical ensemble. The function $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$ at fixed $\boldsymbol{\xi}$ describes the world line of a particle, labelled by the label $\boldsymbol{\xi}$. The number n of variables $\boldsymbol{\xi}$ may be arbitrary, because it is of no importance, how the elements of the statistical ensemble are labelled. However, usually the number of variables $\boldsymbol{\xi}$ is chosen to be equal to the number of variables $\mathbf{x} = \{x^1, x^2, x^3\}$, in order it be possible to resolve the equations

$$\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi}) \tag{7.15}$$

in the form $\boldsymbol{\xi} = \boldsymbol{\xi}(t, \mathbf{x})$ and to use variables $t, \mathbf{x}$ as independent variables (Euler coordinates).

Actions (7.13) and (7.15) generate the same dynamic equations

$$m\frac{d^2\mathbf{x}}{dt^2} = 0 \tag{7.16}$$

Two dynamic systems (7.13) and (7.14) distinguish only in the fact that the dynamic system (7.14) realizes a single-variant description, whereas the statistical ensemble (7.13) realizes a multivariant description, where initial conditions for different elements of the statistical ensemble are different. Dynamic equations (7.16) form a system of *ordinary differential equations*.

If the motion of a single particle $\mathcal{S}_{\mathrm{st}}$ is multivariant (stochastic), the dynamic equations for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ cease to be ordinary differential equations. They become to be partial differential equations, which cannot be reduced to the system of ordinary differential equations. In this case one cannot obtain dynamic equations for a single particle $\mathcal{S}_{\mathrm{st}}$, although there are dynamic equations for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$.

For instance, let us consider the action of the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}[\mathbf{x},\mathbf{u}] = \int\int_{V_\xi} \left\{\frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\} dt d\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{7.17}$$

The dependent variable $\mathbf{x} = \mathbf{x}(t,\boldsymbol{\xi})$ describes the regular component of the particle motion. The dependent variable $\mathbf{u} = \mathbf{u}(t,\mathbf{x})$ describes the mean value of the stochastic velocity component, $\hbar$ is the quantum constant. Operator

$$\boldsymbol{\nabla} = \left\{\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right\} \tag{7.18}$$

is operator in the space of coordinates $\mathbf{x} = \{x^1, x^2, x^3\}$.

To obtain the action functional for $\mathcal{S}_{\mathrm{st}}$ from the action (7.17) for $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, we should omit integration over $\boldsymbol{\xi}$ in (7.17), as it follows from comparison of (7.13) and (7.14). We obtain

$$\mathcal{A}_{\mathcal{S}_{\mathrm{st}}}[\mathbf{x},\mathbf{u}] = \int \left\{\frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\} dt, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{7.19}$$

where $\mathbf{x} = \mathbf{x}(t)$ and $\mathbf{u} = \mathbf{u}(t,\mathbf{x})$ are dependent dynamic variables. The action functional (7.19) is not well defined (for $\hbar \neq 0$), because the operator $\boldsymbol{\nabla}$ is defined in some 3-dimensional vicinity of point $\mathbf{x}$, but not at the point $\mathbf{x}$ itself. As far as the action functional (7.19) is not well defined, one cannot obtain dynamic equations for $\mathcal{S}_{\mathrm{st}}$. By definition it means that the motion of the particle $\mathcal{S}_{\mathrm{st}}$ is multivariant (stochastic). Setting $\hbar = 0$ in (7.19), we transform the action (7.19) into the action (7.13), because in this case $\mathbf{u} = 0$ in virtue of dynamic equations.

Let us return to the action (7.17) and obtain dynamic equations for the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ of physical systems $\mathcal{S}_{\mathrm{st}}$. Variation of (7.17) with respect to $\mathbf{u}$ gives

$$\begin{aligned}
\delta\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}\left[\mathbf{x},\mathbf{u}\right] &= \int\int_{V_\xi}\left\{m\mathbf{u}\delta\mathbf{u}-\frac{\hbar}{2}\boldsymbol{\nabla}\delta\mathbf{u}\right\}dtd\boldsymbol{\xi} \\
&= \int\int_{V_\mathbf{x}}\left\{m\mathbf{u}\delta\mathbf{u}-\frac{\hbar}{2}\boldsymbol{\nabla}\delta\mathbf{u}\right\}\frac{\partial\left(\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^1,x^2,x^3\right)}dtd\mathbf{x} \\
&= \int\int_{V_\mathbf{x}}\delta\mathbf{u}\left\{m\mathbf{u}\rho+\frac{\hbar}{2}\boldsymbol{\nabla}\rho\right\}dtd\mathbf{x}-\int\oint\frac{\hbar}{2}\rho\delta\mathbf{u}dtd\mathbf{S}
\end{aligned}$$

where

$$\rho=\frac{\partial\left(\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^1,x^2,x^3\right)}=\left(\frac{\partial\left(x^1,x^2,x^3\right)}{\partial\left(\xi_1,\xi_2,\xi_3\right)}\right)^{-1} \tag{7.20}$$

We obtain the following dynamic equation

$$m\rho\mathbf{u}+\frac{\hbar}{2}\boldsymbol{\nabla}\rho=0, \tag{7.21}$$

Variation of (7.17) with respect to $\mathbf{x}$ gives

$$m\frac{d^2\mathbf{x}}{dt^2}=\boldsymbol{\nabla}\left(\frac{m}{2}\mathbf{u}^2-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right) \tag{7.22}$$

Here d/dt means the substantial derivative with respect to time t

$$\frac{dF}{dt}\equiv\frac{\partial\left(F,\xi_1,\xi_2,\xi_3\right)}{\partial\left(t,\xi_1,\xi_2,\xi_3\right)}$$

Resolving (7.21) with respect to $\mathbf{u}$, we obtain the equation

$$\mathbf{u}=-\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho, \tag{7.23}$$

which reminds the expression for the mean velocity of the Brownian particle with the diffusion coefficient $D=\hbar/2m$.

Eliminating the velocity $\mathbf{u}$ from dynamic equations (7.22) and (7.23) and going to independent Eulerian variables $t,\mathbf{x}$, we obtain the dynamic equations of the hydrodynamic type for the mean motion of the stochastic particle $\mathcal{S}_{\mathrm{st}}$

$$m\frac{d^2\mathbf{x}}{dt^2}=m\left(\frac{\partial\mathbf{v}}{\partial t}+\left(\mathbf{v}\boldsymbol{\nabla}\right)\mathbf{v}\right)=-\boldsymbol{\nabla}U_{\mathrm{B}},\qquad\frac{\partial\rho}{\partial t}+\boldsymbol{\nabla}\left(\rho\mathbf{v}\right)=0 \tag{7.24}$$

where $\mathbf{v}=\frac{d\mathbf{x}}{dt}$ and U_{B} is the Bohm potential [6]

$$U_{\mathrm{B}}=U\left(\rho,\boldsymbol{\nabla}\rho,\boldsymbol{\nabla}^2\rho\right)=\frac{\hbar^2}{8m}\frac{\left(\boldsymbol{\nabla}\rho\right)^2}{\rho^2}-\frac{\hbar^2}{4m}\frac{\boldsymbol{\nabla}^2\rho}{\rho} \tag{7.25}$$

In the case of the irrotational flow the hydrodynamic equations (7.24) are equivalent to the Schrödinger equations [6]. Thus, the action (7.17) and dynamic equations (7.24) describe correctly multivariant motion of particles of statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$. But it remains unclear, how the form of the action (7.17) is connected with the multivariant space-time geometry. This question was investigated in [5]. It was shown that for agreement of the space-time geometry with the action (7.17) the distortion $d=\lambda_0^2$ in the world function (1.3) should be chosen in the form

$$d=\lambda_0^2=\frac{\hbar}{2bc} \tag{7.26}$$

where $\hbar$ is the quantum constant, c is the speed of the light and b is the constant from the relation (7.11), connecting the geometric mass μ with the usual mass m of the particle.

Here we suggest only some simple arguments for explanation of the connection of the world function of the space-time and the action (7.17), describing motion the ensemble of free particles.

As it follows from the relation (7.10), the particle velocity, described by the link $\mathcal{T}_{[P_1P_2]}$, has two components. One component $\mathbf{v}_{\rm reg}=\mathbf{p}/m$ is regular. It is determined by the particle momentum $\mathbf{p}$. Other component of velocity $\mathbf{v}_{\rm st}$ is conditioned by the random walk of the particle. Its average value $\langle\mathbf{v}_{\rm st}\rangle$ depends on the state of the whole ensemble. It has the form

$$\langle\mathbf{v}_{\rm st}\rangle=-\alpha r_{\min}c\theta_{\rm M}\boldsymbol{\nabla}\log\rho,\qquad r_{\min}=\sqrt{\frac{3}{2}}\lambda_0,\qquad \theta_{\rm M}=\sqrt{2}\frac{\lambda_0}{\mu} \tag{7.27}$$

where α is some real number of the order of 1 and ρ is the density of particles in the statistical ensemble.

On one hand, it follows from the action (7.17), which gives the true description of the mean particle motion, that the mean stochastic velocity can be presented in the form (7.23). On the other hand, comparison of relations (7.27) and (7.23) leads to result (7.26). It is important, that constant b does not appear in the action (7.17), and one cannot determine the value of b as well the value of $q=\lambda_0^2$ experimentally. Thus, for explanation of quantum effects it is important the fact of the multivariance existence, but not its numerical value.

8 Concluding remarks

The non-Euclidean method of the generalized geometry construction admits one to construct all possible generalized geometries. These geometries may be continuous or discrete, They may have alternating dimension, or have no dimension at all. The non-Euclidean method deals only with the world function, which is the only essential characteristic of geometry. The non-Euclidean method does not use coordinate description, and there is no necessity to take into account and remove arbitrariness, connected with a usage of coordinates.

The non-Euclidean method does not need a separation of geometrical propositions into axioms and theorems. It does not need a test of the axioms consistency, connected with this separation. The non-Euclidean method admits one to discover the property of multivariance, which is very general property of generalized geometries. The multivariance appears to be a very important property of the space-time geometry, responsible for quantum effects. Existence of multivariance dictates a new revision of the space-time geometry. The multivariance of the space-time geometry admits one to move along the way of the further physics geometrization. The particle mass appears to be a geometrical characteristic of a particle. It becomes to put the question on existence of geometrical objects. In particular, it becomes to be possible to consider the confinement problem as the geometrical problem of the complicated object existence, but not as a dynamical problem of the pointlike particles confinement inside a restricted volume.

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