Logical reloading. What is it and what is a profit from it?

Yuri A. Rylov

Institute for Problems in Mechanics, Russian Academy of Sciences
101-1,Vernadskii Ave., Moscow, 119526, Russia
email: rylov@ipmnet.ru
Web site: $http://rsfq1.physics.sunysb.edu/\tilde{}rylov/yrylov.htm$
or mirror Web site:
$http://gasdyn-ipm.ipmnet.ru/\tilde{}rylov/yrylov.htm$

Abstract

Logical reloading is a replacement of basic statements of a conception by equivalent statements of the same conception. The logical reloading does not change the conception, but it changes the mathematical formalism and changes results of this conception generalization. In the paper two examples of the logical reloading are considered. (1) Generalization of the deterministic particle dynamics on the case of the stochastic particle dynamics. As a result the unified formalism for description of particles of all kinds appears. This formalism admits one to explain freely quantum dynamics in terms of the classical particle dynamics. In particular, one discovers κ-field responsible for pair production. (2) Generalization of the proper Euclidean geometry which contains such space-time geometries, where free particles move stochastically. As a result such a conception of elementary particle dynamics arises, where one can investigate the elementary particles arrangement, but not only systematize elementary particles, ascribing quantum numbers to them. Besides, one succeeds to expand the general relativity on the non-Riemannian space-time geometries.

1 Introduction

Logical reloading is a logical operation. It is a replacement of basic statements of a conception by another equivalent basic statements. A logical reloading does not change the conception, because new basic concepts are equivalent to the old basic concepts.

Let us consider some logical construction $\mathcal{C}$, which contains basic statements (axioms) $a = \{a_1, a_2, ...a_n\}$ and set $c = \{c_1, c_2, ...\} = \text{cor}\{a\}$ of corollaries of axioms

a. Then the logical construction $\mathcal{C} = \{a, c\}$. Let the same logical construction $\mathcal{C}$ can be formulated in the form $\mathcal{C} = \{A, C\}$, where $A = \{A_1, A_2, ...A_k\}$ is another set of axioms of $\mathcal{C}$ and $C = \{C_1, C_2, ...\} = \mathrm{cor}\,\{A\}$ is a set of corollaries of A. It is evident that logical constructions $\{a, c\}$ and $\{A, C\}$ are equivalent. Replacement of axioms a by axioms A is called a logical reloading. The logical reloading is a logical operation which does not change the logical construction $\mathcal{C}$. However a generalization $\mathcal{C}_{\mathrm{gen}}$ of the logical construction $\mathcal{C}$ depends, generally speaking, on choice of axioms, because $\mathcal{C}'_{\mathrm{gen}} = \{a_{\mathrm{gen}}, \mathrm{cor}\,(a_{\mathrm{gen}})\}$ and $\mathcal{C}''_{\mathrm{gen}} = \{A_{\mathrm{gen}}, \mathrm{cor}\,(A_{\mathrm{gen}})\}$ are different logical constructions, generally speaking. The generalization of axioms a must be such ones, that the new axioms a_{gen} were consistent. The more number n of axioms, the difficult to eliminate inconsistency between them. If the logical construction may be made monistic, and there is only one basic concept or quantity, the problem of inconsistency does not arise. Thus, the logical reloading is a very simple change of a conception, which after generalization leads to fundamental change of existing conception.

Here we shall consider two examples of the logical reloading leading to fundamental change of existing conception: (1) logical reloading in dynamics of deterministic particles and (2) logical reloading in the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. In the first case one succeeded to construct unified formalism for dynamics of deterministic, stochastic and quantum particles. It appears, that quantum particles are stochastic particles which can be described by methods of classical dynamics. It appears that quantum principles are not prime physical principles. Besides, the unified formalism admits one to realize a more detailed description of quantum phenomena. In particular, it appears that elementary particles generate a force field (κ-field) responsible for pair production. The unified method admits one to investigate the elementary particle arrangement but not only to ascribe different quantum numbers to different elementary particles, as it is made in the contemporary theory of elementary particles.

In the second case after logical reloading in the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ one succeeds to construct the set $\mathcal{S}_{\mathrm{ph}}$ of physical space-time geometries, which contains, in particular, discrete space-time geometries. Riemannian geometries form a small part of $\mathcal{S}_{\mathrm{ph}}$. Extension of the general relativity on the set $\mathcal{S}_{\mathrm{ph}}$ of physical space-time geometries led to conclusion, that the dark holes cannot be formed, because of induced antigravitation [1, 2]. Discrete space-time geometry in microcosm explains freely, why a free motion of elementary particles is stochastic.

It seems very unexpected, why such a simple modification of a of the proper Euclidean geometry as the logical reloading could lead to such fundamental change of the contemporary theory in microcosm and in the cosmos. It is worth to stress that such a results are obtained only at consequent application of the prime physical principles (the quantum principles are not the prime physical principles) and at correction of mistakes in their application. No additional hypotheses were used.

2 Logical reloading in the dynamics of deterministic particles

Conventionally in the dynamics of deterministic particles the single particle is considered as a basic object of dynamics. In the nonrelativistic case the dynamical equations for a single particle have the form

$$m\frac{d^2\mathbf{x}}{dt^2} = \mathbf{F}\left(t, \mathbf{x}\right) \tag{2.1}$$

where m is the particle mass and $\mathbf{F}$ is a force field in the space, where the particle moves. The set of many identical independent particle is known as a statistical ensemble. If the number of particle is infinite, the statistical ensemble may be considered as a continuous medium (gas). Dynamic equations for the statistical ensemble have the form

$$\frac{d\mathbf{v}\left(t, \boldsymbol{\xi}\right)}{dt} = \frac{1}{m}\mathbf{F}\left(t, \mathbf{x}\left(t, \boldsymbol{\xi}\right)\right), \quad \frac{d\mathbf{x}\left(t, \boldsymbol{\xi}\right)}{dt} = \mathbf{v}\left(t, \boldsymbol{\xi}\right) \tag{2.2}$$

where $\boldsymbol{\xi}$ is the Lagrangian coordinate labelling particles of the pure statistical ensemble. Thus, the pure statistical ensemble of deterministic particles can be described in terms of ordinary differential equations. Statistical ensemble, where there is only one particle in infinitesimal volume is called the pure statistical ensemble. It is described by dynamic equations (2.2). Mixture of several pure statistical ensembles form a mixed statistical ensemble, which is described by another dynamic equations. For brevity we shall use the term "statistical ensemble" or the term "ensemble" instead of pure statistical ensemble. In the Euler representation the equations (2.2) take the form

$$\frac{\partial \mathbf{v}}{\partial t} + \left(\mathbf{v}\boldsymbol{\nabla}\right)\mathbf{v} = \frac{1}{m}\mathbf{F}, \quad \frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\left(\rho \mathbf{v}\right) = 0 \tag{2.3}$$

where $\rho = mn$ is the density of the gas, and n is the particle concentration.

Comparing equations (2.1) and (2.2) one can conclude that equations (2.2) can be obtained from (2.1) and vice versa equations (2.1) can be obtained from (2.2). It means that the pure statistical ensemble of deterministic particles may be considered as the basic object of dynamics of deterministic particle. In this case the dynamic equations (2.1) for a single particle are a corollary (special case) of dynamic equations (2.2) for a pure statistical ensemble.

For a single stochastic (nondeterministic) particle there are no dynamic equations, but they exist for a pure statistical ensemble of stochastic particles. They have the form

$$\frac{\partial \mathbf{v}}{\partial t} + \left(\mathbf{v}\boldsymbol{\nabla}\right)\mathbf{v} = -\frac{1}{\rho}\boldsymbol{\nabla}p + \frac{1}{m}\mathbf{F}, \quad \frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\left(\rho \mathbf{v}\right) = 0 \tag{2.4}$$

where $p = p\left(t, \mathbf{x}\right)$ is the pressure in the continuous medium (pure statistical ensemble of stochastic particles). The form of the function $p\left(t, \mathbf{x}\right)$ depends on the character of stochasticity. Equations (2.4) cannot be reduced to a system of ordinary differential equations. It means that there are no dynamic equations for a single stochastic particle.

Thus, if the basic object of the particle dynamics is a pure statistical ensemble, one describes the particle dynamics by dynamic equation (2.4). Dynamic equations for deterministic particles and for stochastic ones differ only in the form of the function $p(t, \mathbf{x})$, which vanishes in the case of deterministic particles. As a result one obtains an unified formalism for description of deterministic and stochastic particles. Using this formalism one may show that quantum particles are stochastic particles, which can be described by this unified formalism. One may find out what is the wave function, and from where it appears.

3 Quantum particles as stochastic particles

Let us consider a statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm st}]$ of stochastic particles $\mathcal{S}_{\rm st}$ described by the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm st}]}[\mathbf{x}, \mathbf{u}] = \int \int_{V_\xi} \left\{ \frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u} - V(\mathbf{x}) \right\} \rho_0(\boldsymbol{\xi})\, dt d\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \quad (3.1)$$

The variable $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$ describes the regular component of the particle motion. The variable $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ describes the mean value of the stochastic velocity component, $\hbar$ is the quantum constant. $V(\mathbf{x})$ is some potential of an external force field. The second term in (3.1) describes the kinetic energy of the stochastic velocity component. The third term describes interaction between the stochastic component $\mathbf{u}(t, \mathbf{x})$ and the regular component $\mathbf{x}(t, \boldsymbol{\xi})$. The operator

$$\boldsymbol{\nabla} = \left\{ \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right\} \quad (3.2)$$

is defined in the space of coordinates $\mathbf{x}$. Formally the action (3.1) may be considered as a set of deterministic particles moving in the external field $V(\mathbf{x})$ and interacting between themselves via some force field $\mathbf{u}$. The particles are labelled by parameters $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$. Dynamic equations for variables $\mathbf{x}$ and $\mathbf{u}$ are obtained as a result of variation of the action (3.1) with respect to $\mathbf{x}$ and $\mathbf{u}$ respectively.

The action (3.1) describes a flow of some ideal fluid, and it can be described in terms of a wave function, because the wave function is a way of ideal fluid description [3].

After a proper change of variables the action (3.1) is reduced to the form [4]

$$\begin{aligned} \mathcal{A}[\psi, \psi^*] &= \int \left\{ \frac{i\hbar}{2}(\psi^*\partial_0\psi - \partial_0\psi^* \cdot \psi) - \frac{\hbar^2}{2m}\boldsymbol{\nabla}\psi^* \cdot \boldsymbol{\nabla}\psi \right. \\ &\quad \left. + \frac{\hbar^2}{8m}\rho\boldsymbol{\nabla}s_\alpha\boldsymbol{\nabla}s_\alpha - V(x)\,\rho \right\} \mathrm{d}^4x \end{aligned} \quad (3.3)$$

Here $\psi = \binom{\psi_1}{\psi_2}$ is two-component complex wave function, and

$$\rho = \psi^* \psi, \qquad s_\alpha = \frac{\psi^* \sigma_\alpha \psi}{\rho}, \qquad \alpha = 1, 2, 3 \tag{3.4}$$

where σ_α are 2×2 Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{3.5}$$

In the case, when the wave function ψ is one-component, for instance $\psi = \left\{ \begin{smallmatrix} \psi_1 \\ 0 \end{smallmatrix} \right\}$, the quantities $\mathbf{s} = \{s_1, s_2, s_3\}$ are constant ($s_1 = 0, \quad s_2 = 0, \quad s_3 = 1$), the action (3.3) turns into

$$\mathcal{A}[\psi, \psi^*] = \int \left\{ \frac{i\hbar}{2} (\psi^* \partial_0 \psi - \partial_0 \psi^* \cdot \psi) - \frac{\hbar^2}{2m} \boldsymbol{\nabla} \psi^* \cdot \boldsymbol{\nabla} \psi - V(x) \psi^* \psi \right\} \mathrm{d}^4 x \tag{3.6}$$

The dynamic equation, generated by the action (3.6), is the Schrödinger equation

$$i\hbar \partial_0 \psi + \frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 \psi - V(x) \psi = 0 \tag{3.7}$$

Dynamic equation generated by the action (3.3) has the form

$$i\hbar \partial_0 \psi + \frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 \psi + \frac{\hbar^2}{8m} \boldsymbol{\nabla}^2 s_\alpha \cdot (s_\alpha - 2\sigma_\alpha) \psi - \frac{\hbar^2}{4m} \frac{\boldsymbol{\nabla} \rho}{\rho} \boldsymbol{\nabla} s_\alpha \sigma_\alpha \psi - V(x) \psi = 0 \tag{3.8}$$

It describes a rotational flow of the statistical ensemble fluid, whereas the Schrödinger equation (3.7) describes nonrotational flow [5].

Transition from variables $\mathbf{x}(t, \boldsymbol{\xi}), \mathbf{u}(t, \mathbf{x})$ in the action (3.1) to the wave function ψ in the action (3.3) is not algebraic. It includes integration of dynamic equations generated by the action (3.1). Three arbitrary functions $\mathbf{g}(\boldsymbol{\xi}) = \{g^1(\boldsymbol{\xi}), g^2(\boldsymbol{\xi}), g^3(\boldsymbol{\xi})\}$ appear as a result of this integration. The wave function is constructed of these functions $\mathbf{g}(\boldsymbol{\xi})$. Process of this change of variables is not simple. It is described in [4]. But here I shall not go into details of this change of variables. It will be made for the relativistic case in sec.5. Consequence of the fact that the quantum particles are simply stochastic particles, which can be described without a use of quantum principles, is very important, because this circumstance reduces the number of physical essences in the particle dynamics.

On one hand, the logical reloading is an evident logical procedure which does not use any additional suppositions or hypotheses. On the other hand, the logical reloading changes fundamentals of a physical conception, transposing physical concepts and changing their relative significance. In particular, unified formalism for description of deterministic, stochastic and quantum particles admits one to explain quantum principles and quantum essences in terms of classical concepts.

4 Relativistic stochastic particles

Motion of a pointlike relativistic stochastic particle is described by the action

$$\mathcal{A}[x,\kappa] = \int \left\{ -mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k} - \frac{e}{c}A_k\dot{x}^k \right\} d^4\xi, \quad d^4\xi = d\xi_0 d\boldsymbol{\xi} \tag{4.1}$$

$$K = \sqrt{1+\lambda^2\left(\kappa_l\kappa^l + \partial_l\kappa^l\right)}, \qquad \lambda = \frac{\hbar}{mc}, \quad \tau = \boldsymbol{\xi}_0 \tag{4.2}$$

Here $x = \{x^i(\xi_0, \boldsymbol{\xi})\}$, $i = 0,1,2,3$ are dependent variables, describing regular component of the particle motion. The variables $\xi = \{\xi_0, \boldsymbol{\xi}\} = \{\xi_k\}$, $k = 0,1,2,3$ are independent variables, labelling the particles of the statistical ensemble, and $\dot{x}^i \equiv dx^i/d\xi_0$. The quantities $\kappa^l = \left\{\kappa^l(x)\right\}$, $l = 0,1,2,3$ are dependent variables, describing stochastic component of the particle motion, A_k, $k = 0,1,2,3$ is the potential of the external electromagnetic field. Note that the action (3.1) for nonrelativistic stochastic particles is obtained from (4.1), (4.2) in the case, when $\lambda^2\left(\kappa_l\kappa^l + \partial_l\kappa^l\right) \ll 1$ and $|\kappa_0| \ll |\boldsymbol{\kappa}| = |m\mathbf{u}/\hbar|$.

We shall refer to the dynamic system, described by the action (4.1), (4.2) as $\mathcal{S}_{\text{KG}}$, because irrotational flow of $\mathcal{S}_{\text{KG}}$ is described by the Klein-Gordon equation [6]. We present here this transformation to the Klein-Gordon form. Here and farther a summation is produced over repeated Latin indices $(0 \div 3)$ and over Greek indices $(1 \div 3)$.

Dynamic equations generated by the action (4.1), (4.2) are equations of the hydrodynamical type. To present these equations in terms of the wave function, one needs to integrate them in general form. The problem of general integration of four hydrodynamic Euler equations

$$\partial_0\rho + \boldsymbol{\nabla}(\rho\mathbf{v}) = 0 \tag{4.3}$$

$$\partial_0\mathbf{v} + (\mathbf{v}\boldsymbol{\nabla})\mathbf{v} = -\frac{1}{\rho}\boldsymbol{\nabla}p, \qquad p = p(\rho, \boldsymbol{\nabla}\rho) \tag{4.4}$$

seems to be hopeless. It is really so, if the Euler system (4.3), (4.4) is considered to be a complete system of dynamic equations. In fact, the Euler equations (4.3), (4.4) do not form a complete system of dynamic equations, because it does not describe motion of fluid particles along their trajectories. To obtain the complete system of dynamic equations, we should add to the Euler system so called Lin constraints [7]

$$\partial_0\boldsymbol{\xi} + (\mathbf{v}\boldsymbol{\nabla})\boldsymbol{\xi} = 0 \tag{4.5}$$

where $\boldsymbol{\xi} = \boldsymbol{\xi}(t, \mathbf{x}) = \{\xi_1, \xi_2, \xi_3\}$ are three independent integrals of dynamic equations

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(t, \mathbf{x}),$$

describing motion of fluid particles in the given velocity field.

Seven equations (4.3).– (4.5) form the complete system of dynamic equations, whereas four Euler equations (4.3), (4.4) form only a closed subsystem of the complete system of dynamic equations. The wave function is expressed via hydrodynamic potentials $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, which are known also as Clebsch potentials [8, 9].

In general case of arbitrary fluid flow in three-dimensional space the complex wave function ψ has two complex components ψ_1, ψ_2 (or four independent real components)

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \sqrt{\rho} e^{i\varphi} u_1 (\boldsymbol{\xi}) \\ \sqrt{\rho} e^{i\varphi} u_2 (\boldsymbol{\xi}) \end{pmatrix}, \qquad |u_1|^2 + |u_2|^2 = 1 \tag{4.6}$$

The system of hydrodynamic equations in terms of the wave function ψ contains four real dynamic equations of the first order with respect to time derivative [5]. The complete system (4.3) - (4.5) of hydrodynamic equations contains seven dynamic equations of the first order with respect to time derivative. To write dynamic equations (4.3) - (4.5) in terms of the wave function, one needs to integrate them and to reduce their number.

It is impossible to obtain general solution of the Euler system (4.3), (4.4), but one can partially integrate the complete system (4.3) – (4.5), reducing its order to four dynamic equations for the wave function (4.6). Practically it means that one integrates dynamic equations (4.5), where the function $\mathbf{v}(t, \mathbf{x})$ is determined implicitly by equations (4.3), (4.4). Such an integration and reduction of the order of the complete system of dynamic equations appear to be possible, because the system (4.3) – (4.5) has the symmetry group, connected with transformations of the Clebsch potentials

$$\xi_\alpha \rightarrow \tilde{\xi}_\alpha = \tilde{\xi}_\alpha (\boldsymbol{\xi}), \qquad \alpha = 1, 2, 3, \qquad \frac{\partial \left(\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3 \right)}{\partial (\xi_1, \xi_2, \xi_3)} \neq 0 \tag{4.7}$$

5 Transformation of the action to description in terms of the wave function

Let us consider variables $\xi = \xi(x)$ in (4.1) as dependent variables and variables x as independent variables. Let the Jacobian

$$J = \frac{\partial (\xi_0, \xi_1, \xi_2, \xi_3)}{\partial (x^0, x^1, x^2, x^3)} = \det \left|\left|\xi_{i,k}\right|\right|, \qquad \xi_{i,k} \equiv \partial_k \xi_i \equiv \frac{\partial \xi_i}{\partial x^k}, \qquad i, k = 0, 1, 2, 3 \tag{5.1}$$

be considered to be a multilinear function of $\xi_{i,k}$. Then

$$d^4\xi = J d^4 x, \qquad \dot{x}^i \equiv \frac{dx^i}{d\xi_0} \equiv \frac{\partial (x^i, \xi_1, \xi_2, \xi_3)}{\partial (\xi_0, \xi_1, \xi_2, \xi_3)} = J^{-1} \frac{\partial J}{\partial \xi_{0,i}} \tag{5.2}$$

After transformation to dependent variables ξ the action (4.1) takes the form

$$\mathcal{A}[\xi, \kappa] = \int \left\{ -mcK \sqrt{g_{ik} \frac{\partial J}{\partial \xi_{0,i}} \frac{\partial J}{\partial \xi_{0,k}}} - \frac{e}{c} A_k \frac{\partial J}{\partial \xi_{0,k}} \right\} d^4 x, \tag{5.3}$$

$$K = \sqrt{1 + \lambda^2 \left(\kappa_l \kappa^l + \partial_l \kappa^l \right)}, \qquad \lambda = \frac{\hbar}{mc}, \tag{5.4}$$

Now variables ξ and κ are considered as functions of independent variables x.

Let us introduce new variables

$$j^k = \frac{\partial J}{\partial \xi_{0,k}}, \qquad k = 0,1,2,3 \tag{5.5}$$

by means of Lagrange multipliers p_k

$$\mathcal{A}[\xi,\kappa,j,p] = \int \left\{ -mcK\sqrt{g_{ik}j^i j^k} - \frac{e}{c}A_k j^k + p_k \left(\frac{\partial J}{\partial \xi_{0,k}} - j^k \right) \right\} d^4x, \tag{5.6}$$

Variation with respect to ξ_i gives

$$\frac{\delta \mathcal{A}}{\delta \xi_i} = -\partial_l \left(p_k \frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \right) = 0, \qquad i = 0,1,2,3 \tag{5.7}$$

Using identities

$$\frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \equiv J^{-1} \left(\frac{\partial J}{\partial \xi_{0,k}} \frac{\partial J}{\partial \xi_{i,l}} - \frac{\partial J}{\partial \xi_{0,l}} \frac{\partial J}{\partial \xi_{i,k}} \right) \tag{5.8}$$

$$\frac{\partial J}{\partial \xi_{i,l}} \xi_{k,l} \equiv J\delta_k^i, \quad \partial_l \frac{\partial J}{\partial \xi_{i,l}} \equiv 0 \qquad \partial_l \frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \equiv 0 \tag{5.9}$$

one can test by direct substitution that the general solution of linear equations (5.7) has the form

$$p_k = b_0 \left(\partial_k \varphi + g^\alpha (\boldsymbol{\xi}) \partial_k \xi_\alpha \right), \qquad k = 0,1,2,3 \tag{5.10}$$

where $b_0 \neq 0$ is a constant, $g^\alpha(\boldsymbol{\xi})$, $\alpha = 1,2,3$ are arbitrary functions of $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, and φ is the dynamic variable ξ_0, which ceases to be fictitious. Let us substitute (5.10) in (5.6). The term of the form $\partial J/\partial \xi_{0,k} \partial_k \varphi$ is reduced to Jacobian and does not contribute to dynamic equations. The terms of the form $\xi_{\alpha,k} \partial J/\partial \xi_{0,k}$ vanish due to identities (5.9). We obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa, j] = \int \left\{ -mcK\sqrt{g_{ik}j^i j^k} - j^k \pi_k \right\} d^4x, \tag{5.11}$$

where quantities π_k are determined by the relations

$$\pi_k = b_0 \left(\partial_k \varphi + g^\alpha (\boldsymbol{\xi}) \partial_k \xi_\alpha \right) + \frac{e}{c} A_k, \qquad k = 0,1,2,3 \tag{5.12}$$

Integration of (5.7) in the form (5.10) is that integration which admits to introduce a wave function. Note that coefficients in the system of equations (5.7) at derivatives of p_k are constructed of minors of the Jacobian (5.1). It is the circumstance that admits one to produce a formal general integration.

Variation of (5.11) with respect to κ^l gives

$$\frac{\delta \mathcal{A}}{\delta \kappa^l} = -\frac{\lambda^2 mc\sqrt{g_{ik}j^i j^k}}{K} \kappa_l + \partial_l \frac{\lambda^2 mc\sqrt{g_{ik}j^i j^k}}{2K} = 0, \quad \lambda = \frac{\hbar}{mc} \tag{5.13}$$

It can be written in the form

$$\kappa_l = \partial_l \kappa = \frac{1}{2} \partial_l \ln \rho, \qquad e^{2\kappa} = \frac{\rho}{\rho_0} \equiv \frac{\sqrt{j_s j^s}}{\rho_0 K}, \quad \rho = \frac{\sqrt{j_s j^s}}{K} \tag{5.14}$$

where the variable κ is potential of the κ-field κ_i and ρ_0 =const is the integration constant. Substituting (5.4) in (5.14), we obtain dynamic equation for κ

$$\hbar^2 \left(\partial_l \kappa \cdot \partial^l \kappa + \partial_l \partial^l \kappa \right) = m^2 c^2 \frac{e^{-4\kappa} j_s j^s}{\rho_0^2} - m^2 c^2 \tag{5.15}$$

Variation of (5.11) with respect to j^k gives

$$\pi_k = -\frac{mcK j_k}{\sqrt{g_{ls} j^l j^s}} \tag{5.16}$$

or

$$\pi_k g^{kl} \pi_l = m^2 c^2 K^2 \tag{5.17}$$

Substituting $\sqrt{j_s j^s}/K$ from the second equation (5.14) in (5.16), we obtain

$$j_k = -\frac{\rho_0}{mc} e^{2\kappa} \pi_k, \tag{5.18}$$

Now we eliminate the variables j^k from the action (5.11), using relation (5.18) and (5.14). We obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa] = \int \rho_0 e^{2\kappa} \left\{ -m^2 c^2 K^2 + \pi^k \pi_k \right\} d^4 x, \tag{5.19}$$

where π_k is determined by the relation (5.12). Using expression (4.2) for K, the first term of the action (5.19) can be transformed as follows.

$$\begin{aligned} -m^2 c^2 e^{2\kappa} K^2 &= -m^2 c^2 e^{2\kappa} \left(1 + \lambda^2 \left(\partial_l \kappa \partial^l \kappa + \partial_l \partial^l \kappa \right) \right) \\ &= -m^2 c^2 e^{2\kappa} + \hbar^2 e^{2\kappa} \partial_l \kappa \partial^l \kappa - \frac{\hbar^2}{2} \partial_l \partial^l e^{2\kappa} \end{aligned}$$

Let us take into account that the last term has the form of divergence. It does not contribute to dynamic equations and can be omitted. Omitting this term, we obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa] = \int \rho_0 e^{2\kappa} \left\{ -m^2 c^2 + \hbar^2 \partial_l \kappa \partial^l \kappa + \pi^k \pi_k \right\} d^4 x, \tag{5.20}$$

Here π_k is defined by the relation (5.12), where the integration constant b_0 is chosen in the form $b_0 = \hbar$

$$\pi_k = \hbar \left(\partial_k \varphi + g^\alpha (\boldsymbol{\xi}) \partial_k \xi_\alpha \right) + \frac{e}{c} A_k, \qquad k = 0, 1, 2, 3 \tag{5.21}$$

Instead of dynamic variables $\varphi, \boldsymbol{\xi}, \kappa$ we introduce n-component complex function

$$\psi = \{\psi_\alpha\} = \left\{ \sqrt{\rho} e^{i\varphi} w_\alpha (\boldsymbol{\xi}) \right\} = \left\{ \sqrt{\rho_0} e^{\kappa + i\varphi} w_\alpha (\boldsymbol{\xi}) \right\}, \qquad \alpha = 1, 2, \ldots n \tag{5.22}$$

Here w_α are functions of only $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, having the following properties

$$\sum_{\alpha=1}^{\alpha=n} w_\alpha^* w_\alpha = 1, \qquad -\frac{i}{2}\sum_{\alpha=1}^{\alpha=n}\left(w_\alpha^* \frac{\partial w_\alpha}{\partial \xi_\beta} - \frac{\partial w_\alpha^*}{\partial \xi_\beta} w_\alpha\right) = g^\beta\left(\boldsymbol{\xi}\right) \tag{5.23}$$

where $(^*)$ denotes the complex conjugation. The number n of components of the wave function ψ depends on the functions $g^\beta\left(\boldsymbol{\xi}\right)$. The number n is chosen in such a way, that equations (5.23) have a solution. Then we obtain

$$\psi^*\psi \equiv \sum_{\alpha=1}^{\alpha=n} \psi_\alpha^*\psi_\alpha = \rho = \rho_0 e^{2\kappa}, \qquad \partial_l \kappa = \frac{\partial_l\left(\psi^*\psi\right)}{2\psi^*\psi} \tag{5.24}$$

$$\pi_k = -\frac{i\hbar\left(\psi^*\partial_k\psi - \partial_k\psi^*\cdot\psi\right)}{2\psi^*\psi} + \frac{e}{c}A_k, \qquad k = 0,1,2,3 \tag{5.25}$$

Substituting relations (5.24), (5.25) in (5.20), we obtain the action, written in terms of the wave function ψ

$$\begin{aligned}\mathcal{A}\left[\psi,\psi^*\right] = & \int\left\{\left[\frac{i\hbar\left(\psi^*\partial_k\psi - \partial_k\psi^*\cdot\psi\right)}{2\psi^*\psi} - \frac{e}{c}A_k\right]\left[\frac{i\hbar\left(\psi^*\partial^k\psi - \partial^k\psi^*\cdot\psi\right)}{2\psi^*\psi} - \frac{e}{c}A^k\right]\right. \\ & \left. + \hbar^2\frac{\partial_l\left(\psi^*\psi\right)\partial^l\left(\psi^*\psi\right)}{4\left(\psi^*\psi\right)^2} - m^2c^2\right\}\psi^*\psi d^4x\end{aligned} \tag{5.26}$$

Let us use the identity

$$\begin{aligned}& \frac{\left(\psi^*\partial_l\psi - \partial_l\psi^*\cdot\psi\right)\left(\psi^*\partial^l\psi - \partial^l\psi^*\cdot\psi\right)}{4\psi^*\psi} + \partial_l\psi^*\partial^l\psi \\ \equiv & \frac{\partial_l\left(\psi^*\psi\right)\partial^l\left(\psi^*\psi\right)}{4\psi^*\psi} + \frac{g^{ls}}{2}\psi^*\psi\sum_{\alpha,\beta=1}^{\alpha,\beta=n} Q_{\alpha\beta,l}^* Q_{\alpha\beta,s}\end{aligned} \tag{5.27}$$

where

$$Q_{\alpha\beta,l} = \frac{1}{\psi^*\psi}\begin{vmatrix}\psi_\alpha & \psi_\beta \\ \partial_l\psi_\alpha & \partial_l\psi_\beta\end{vmatrix}, \qquad Q_{\alpha\beta,l}^* = \frac{1}{\psi^*\psi}\begin{vmatrix}\psi_\alpha^* & \psi_\beta^* \\ \partial_l\psi_\alpha^* & \partial_l\psi_\beta^*\end{vmatrix} \tag{5.28}$$

Then we obtain

$$\begin{aligned}\mathcal{A}\left[\psi,\psi^*\right] = & \int\left\{\left(i\hbar\partial_k + \frac{e}{c}A_k\right)\psi^*\left(-i\hbar\partial^k + \frac{e}{c}A^k\right)\psi - m^2c^2\psi^*\psi\right. \\ & \left. + \frac{\hbar^2}{2}\sum_{\alpha,\beta=1}^{\alpha,\beta=n} g^{ls}Q_{\alpha\beta,l}Q_{\alpha\beta,s}^*\psi^*\psi\right\}d^4x\end{aligned} \tag{5.29}$$

Let us consider the case of irrotational flow, when $g^\alpha\left(\boldsymbol{\xi}\right) = 0$. In this case $w_1 = 1$, $w_2 = 0$, and the function ψ has only one component. It follows from (5.28), that $Q_{\alpha\beta,l} = 0$. Then we obtain instead of (5.29)

$$\mathcal{A}\left[\psi,\psi^*\right] = \int\left\{\left(i\hbar\partial_k + \frac{e}{c}A_k\right)\psi^*\left(-i\hbar\partial^k + \frac{e}{c}A^k\right)\psi - m^2c^2\psi^*\psi\right\}d^4x \tag{5.30}$$

Variation of the action (5.30) with respect to ψ^* generates the Klein-Gordon equation

$$\left(-i\hbar\partial_k + \frac{e}{c}A_k\right)\left(-i\hbar\partial^k + \frac{e}{c}A^k\right)\psi - m^2c^2\psi = 0 \qquad (5.31)$$

Thus, description in terms of the Klein-Gordon equation is a special case of the stochastic particles description by means of the action (4.1), (4.2).

In the case, when the fluid flow is rotational, and the wave function ψ is two-component, the identity (5.27) takes the form

$$\begin{aligned} & \frac{\left(\psi^*\partial_l\psi - \partial_l\psi^*\cdot\psi\right)\left(\psi^*\partial^l\psi - \partial^l\psi^*\cdot\psi\right)}{4\rho} - \frac{\left(\partial_l\rho\right)\left(\partial^l\rho\right)}{4\rho} \\ \equiv \quad & -\partial_l\psi^*\partial^l\psi + \frac{1}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\rho \end{aligned} \qquad (5.32)$$

where 3-vector $\mathbf{s} = \{s_1, s_2, s_3, \}$ is defined by the relations

$$\rho = \psi^*\psi, \qquad s_\alpha = \frac{\psi^*\sigma_\alpha\psi}{\rho}, \qquad \alpha = 1,2,3 \qquad (5.33)$$

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \qquad \psi^* = \left(\psi_1^*, \psi_2^*\right), \qquad (5.34)$$

and Pauli matrices $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ have the form (3.5) Note that 3-vectors $\mathbf{s}$ and $\boldsymbol{\sigma}$ are vectors in the space V_ξ of the Clebsch potentials $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$. They transform as vectors at the transformations (4.7)

In general, transformations of Clebsch potentials $\boldsymbol{\xi}$ and those of coordinates $\mathbf{x}$ are independent. However, the action (5.26) does not contain any reference to the Clebsch potentials $\boldsymbol{\xi}$ and transformations (4.7) of $\boldsymbol{\xi}$. If we consider only linear transformations of space coordinates $\mathbf{x}$

$$x^\alpha \to \tilde{x}^\alpha = b^\alpha + \omega^\alpha_{.\beta}x^\beta, \qquad \alpha = 1,2,3 \qquad (5.35)$$

nothing prevents from accompanying any transformation (5.35) with the similar transformation

$$\xi_\alpha \to \tilde{\xi}_\alpha = b^\alpha + \omega^\alpha_{.\beta}\xi_\beta, \qquad \alpha = 1,2,3 \qquad (5.36)$$

of Clebsch potentials $\boldsymbol{\xi}$. The formulas for linear transformation of vectors and spinors in V_x do not contain the coordinates $\mathbf{x}$ explicitly, and one can consider vectors and spinors in V_ξ as vectors and spinors in V_x, provided we consider linear transformations (5.35), (5.36) always together.

Using identity (5.32), we obtain from (5.26)

$$\mathcal{A}\left[\psi, \psi^*\right] = \int \left\{\left(i\hbar\partial_k + \frac{e}{c}A_k\right)\psi^*\left(-i\hbar\partial^k + \frac{e}{c}A^k\right)\psi - m^2c^2\rho - \frac{\hbar^2}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\rho\right\} d^4x \qquad (5.37)$$

Dynamic equation, generated by the action (5.37), has the form

$$\begin{aligned} & \left(-i\hbar\partial_k + \frac{e}{c}A_k\right)\left(-i\hbar\partial^k + \frac{e}{c}A^k\right)\psi - \left(m^2c^2 + \frac{\hbar^2}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\right)\psi \\ = \quad & -\hbar^2\frac{\partial_l\left(\rho\partial^l s_\alpha\right)}{2\rho}\left(\sigma_\alpha - s_\alpha\right)\psi \end{aligned} \qquad (5.38)$$

The gradient of the unit 3-vector $\mathbf{s}=\{s_1,s_2,s_3\}$ describes rotational component of the fluid flow. If $\mathbf{s}=\text{const}$, the dynamic equation (5.38) turns to the conventional Klein-Gordon equation (5.31).

6 Several identical relativistic particles

Let us consider N identical relativistic stochastic particles, whose electric charge vanishes. They are described by the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\text{st}}]}\left[X,\kappa,A\right]=\sum_{A=1}^{A=N}\int\limits_{V_{\boldsymbol{\xi}}}L_{(A)}\left(x_{(A)}\left(\tau,\boldsymbol{\xi}\right)\right)d\tau d\boldsymbol{\xi} \tag{6.1}$$

$$X=\left\{x_{(1)},x_{(2)},...x_{(N)}\right\},\quad x_{(A)}=\left\{x_{(A)}^0,x_{(A)}^1,x_{(A)}^2,x_{(A)}^3\right\},\quad A=1,2,...N \tag{6.2}$$

Here an index in brackets means the number of a particle.

$$L_{(A)}\left(x_{(A)}\left(\tau,\boldsymbol{\xi}\right)\right)=-M_{(A)}\left(x_{(A)}\right)c\sqrt{g_{ik}\dot{x}_{(A)}^i\dot{x}_{(A)}^k},\quad A=1,2,...N \tag{6.3}$$

$$\dot{x}_{(A)}^i=\frac{dx_{(A)}^i}{d\tau},\quad x_{(A)}=x_{(A)}\left(\tau,\boldsymbol{\xi}\right) \tag{6.4}$$

$$M_{(A)}=M_{(A)}\left(x_{(A)}\right)=\sqrt{m^2+\left(\frac{\hbar}{c}\right)^2\left(g_{kl}\kappa^k\left(x_A\right)\kappa^l\left(x_A\right)+\frac{\partial}{\partial x_{(A)}^k}\kappa^k\left(x_A\right)\right)},\quad A=1,2,...N \tag{6.5}$$

$M_{(A)}$ is the effective mass of the Ath particle, and m is its usual mass.

Describing these particles in terms of the wave function, one obtains

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\text{st}}]}\left[\psi,\psi^*\right]=\sum_{A=1}^{A=N}\int\limits_{V_{x_{(A)}}}L_{(A)}\left(\psi\left(X\right),\psi^*\left(X\right)\right)d^4x_{(A)} \tag{6.6}$$

$$L_{(A)}\left(\psi\left(X\right),\psi^*\left(X\right)\right)=\frac{\partial\psi^*}{\partial x_{(A)}^k}g^{ik}\frac{\partial\psi}{\partial x_{(A)}^i}-m^2c^2\psi^*\psi \tag{6.7}$$

Dynamic equations have the form

$$g^{ik}\frac{\partial^2\psi}{\partial x_{(A)}^i\partial x_{(A)}^k}+m^2c^2\psi=0,\quad A=1,2,...N \tag{6.8}$$

Solution has the form

$$\psi\left(X\right)=\prod_{A=1}^{A=N}\psi_{(A)}\left(x_{(A)}\right) \tag{6.9}$$

where $\psi_{(A)}\left(x_{(A)}\right)$ is the wave function of Ath particle. It satisfies the equation

$$g^{ik}\frac{\partial^2\psi_{(A)}(x)}{\partial x^i\partial x^k}+m^2c^2\psi_{(A)}(x)=0,\quad A=1,2,...N \tag{6.10}$$

After symmetrization one obtains

$$\psi(X)=\sum_{\substack{\text{permutations}\\ x_{(A)}\longleftrightarrow x_{(B)}}}\prod_{A=1}^{A=N}\psi_{(A)}\left(x_{(A)}\right) \tag{6.11}$$

The sum symbol means the sum of all permutations of arguments $x_{(A)}$. All particles, described by relations (6.11), (6.10) are considered to be noninteracting. At such a description the κ-field $\kappa\left(x_{(A)}\right)$ in (6.1) - (6.3) is considered to be an internal field of the Ath particle. This field is considered as an attribute of the wave function $\psi_{(A)}$. The wave function $\psi_{(A)}$ "assimilates" the κ-field $\kappa\left(x_{(A)}\right)$.

In reality the action (6.1) - (6.5) describes identical stochastic particles interacting via the κ-field, which is a usual force field. A charged particle has the Coulomb electric field which is deformed at the particle motion. The Coulomb field is an external electromagnetic field for other charged particles. It acts on other charged particles. As a result the electromagnetic interaction between charged particles arises.

In a like way any particle generates κ-field, which is an external κ-field for other particles. The external κ-field changes the particle mass. This change of the particle mass may be so strong that the particle world line changes its direction in time. This turn of the world line in time may be interpreted as a pair production or as a pair annihilation [10]. Thus, the κ-field is a force field responsible for pair production.

Dynamic equation for the κ-field of a single particle has the form

$$\left(m^2c^2+\hbar^2g^{kl}\frac{\partial^2}{\partial x^k\partial x^l}\right)e^{\kappa}=\frac{m^2c^2j_sj^s}{\exp(3\kappa)} \tag{6.12}$$

where κ is potential of the κ-field $\kappa_l=g_{kl}\kappa^l=\partial_l\kappa$ and j^k is the 4-current of particles. The κ-field of the particle at rest has the form

$$e^{\kappa}=\left\{\begin{array}{c}\sqrt[4]{j_sj^s},\ \text{if } r<r_0\\ \sqrt[4]{j_sj^s}\frac{e^{-r/\lambda}}{r},\ \text{if } r>r_0\end{array}\right. \tag{6.13}$$

where r_0 ($r_0\ll\lambda=\frac{\hbar}{mc}$) is radius of the space region, where the particle is located. In the region, where the particles are absent ($j^s=0$), e^{κ} satisfies the linear equation

$$\left(m^2c^2+\hbar^2g^{kl}\partial_k\partial_l\right)e^{\kappa}=0 \tag{6.14}$$

The whole expression for the κ-field generated by N identical particles has the form

$$\kappa(X)=\frac{1}{2}\sum_{A=1}^{A=N}\log\frac{m\sqrt{j_{(A)s}\left(x_{(A)}\right)j_{(A)}^s\left(x_{(A)}\right)}}{M_{(A)}\left(x_{(A)}\right)} \tag{6.15}$$

$$X=\left\{x_{(1)},x_{(2)},...x_{(N)}\right\},\quad x_{(A)}=\left\{x_{(A)}^{0},x_{(A)}^{1},x_{(A)}^{2},x_{(A)}^{3}\right\},\quad A=1,2,...N \tag{6.16}$$

where $j_{(A)}^{k}$ is 4-current generated by the Ath particle. Expression (6.15) for κ is symmetric with respect permutation of any arguments $x_{(A)}$ and $x_{(B)}$. According to (6.5) the effective mass $M_{(A)}$ in (6.15) depends on $\kappa\left(x_{(A)}\right)$ and its derivatives. Dynamic equations for the Ath particle 4-velocity $v_{(A)}^{k}$ and for the κ-field are obtained from the action (6.1) – (6.5). They have the form

$$v_{(A)}^{k}\frac{\partial v_{(A)i}}{\partial x_{(A)}^{k}}+\left(v_{(A)}^{k}v_{(A)i}-\delta_{i}^{k}v_{(A)s}v_{(A)}^{s}\right)\frac{\partial}{\partial x_{(A)}^{k}}\left(\log K_{(A)}\left(x_{(A)}\right)\right)=0,\quad A=1,2,...N$$

or

$$v_{(A)}^{k}\frac{\partial v_{(A)i}}{\partial x_{(A)}^{k}}+\left(v_{(A)}^{k}v_{(A)i}-\delta_{i}^{k}v_{(A)s}v_{(A)}^{s}\right)$$
$$\times\frac{\partial}{\partial x_{(A)}^{k}}\log\sqrt{\left(1+\lambda^{2}w^{-1}\left(x_{(A)}\right)g^{ls}\frac{\partial^{2}w\left(x_{(A)}\right)}{\partial x_{(A)}^{l}\partial x_{(A)}^{s}}\right)}=0,\quad A=1,2,...N \tag{6.17}$$

$$\prod_{B=1}^{B=N}\left(\left(1+\lambda^{2}g^{kl}\frac{\partial^{2}}{\partial x_{(B)}^{k}\partial x_{(B)}^{l}}\right)w\left(x_{B}\right)\right)$$
$$=\;w^{-4}\left(x_{(A)}\right)\prod_{B=1}^{B=N}\left(j_{(B)s}\left(x_{(B)}\right)j_{(B)}^{s}\left(x_{(B)}\right)w\left(x_{(B)}\right)\right),\quad A=1,2,...N \tag{6.18}$$

where

$$v_{(A)}^{k}\left(x_{(A)}\right)=\frac{j_{(A)}^{k}\left(x_{(A)}\right)}{\sqrt{j_{(A)s}\left(x_{(A)}\right)j_{(A)}^{s}\left(x_{(A)}\right)}},\quad w\left(x_{(A)}\right)=e^{\kappa\left(x_{(A)}\right)},\quad A=1,2,...N \tag{6.19}$$

N dynamic equations (6.18) are rather unusual. Differential part of them is the same. It consists of differential parts of Klein-Gordon equations. However, the part describing interaction of different particles differs from conventional representation about the particle interaction.

If one writes any of $L_{(A)}$, defined by (6.3) in terms of wave functions, one obtains (6.6), (6.7) instead (6.1) - (6.5). One does not obtain interaction of particles between themselves via κ-field. Absence of interaction is connected with the fact, that the wave function is obtained from the consideration of a single particle. At the introduction of the wave function one produces integration of equation (5.13) in the form of (5.14). This integration generate the quantity ρ_0 which is an absolute constant. At the consideration of many particles in the form of (6.1) - (6.5) a like integration generates N quantities $\rho_{(A)}$, which are relative constants in the sense that $\partial\rho_{0(A)}/\partial x_{(A)}^{i}=0$. However, $\rho_{0(A)}$ may depend on $x_{(B)}$ $B\neq A$. This dependence on coordinates of other particles generates interaction of particles via κ-field.

The energy-momentum tensor T^{ik} have the form

$$T^{ik}=\sum_{A=1}^{A=N}M_{(A)}\left(x_{(A)}\right)c\left(\begin{array}{c}\frac{\dot{x}_{(A)}^{i}\dot{x}_{(A)}^{k}}{\sqrt{\dot{x}_{(A)}^{s}\dot{x}_{(A)s}}}\\ +\left(\frac{\hbar}{c}\right)^{2}\frac{\sqrt{\dot{x}_{(A)}^{s}\dot{x}_{(A)s}}\left(\kappa^{i}(x_{A})\kappa^{k}(x_{A})+\frac{1}{2}\frac{\partial}{\partial x_{(A)i}}\kappa^{k}(x_{A})+\frac{1}{2}\frac{\partial}{\partial x_{(A)k}}\kappa^{i}(x_{A})\right)}{M_{(A)}^{2}\left(x_{(A)}\right)}\end{array}\right) \tag{6.20}$$

The first term describes the energy-momentum of the particle itself, whereas the second one describes the energy-momentum of the κ-field. Near the returning point of the world line $M_{(A)}\left(x_{(A)}\right)\rightarrow 0$ and the first term vanishes. On the contrary, the second term increases, and all energy concentrates in the κ-field.

In the quantum field theory the κ-field is incorporated in the wave function, and researchers do not know about existence of the κ-field. The reason of pair production is a mystery for researches. Nevertheless it is used to think that practically any nonlinear term added to the Klein-Gordon equation may produce pair production [11, 12, 13, 14]. Unfortunately, it is not so, because the force field generating the pair production is to be very special. It must change the effective particle mass. It must appear under radical in the expression for effective mass (6.5) as the κ-field, because at the moment of pair production the effective mass vanishes, and component p_0 of canonical 4-momentum p_k changes its sign.

Why do researchers believe in nonlinear equations as a reason of the pair production? The answer is rather unexpected. There is a mistake in the procedure of the second quantization of the relativistic scalar field. Corollaries of this mistake imitate the pair production. The particles are produced in the form of pairs particle - antiparticle. The reason of such a situation lies in the fact that particle and antiparticle are not independent dynamical objects. They are two different states of emlon. The term "emlon" is a perusal of abbreviation ML, that means an abbreviation of Russian term which means "world line". In dynamics of relativistic particles the world line (emlon) is a main object of dynamics. Particle and antiparticle are two different states of the emlon. The two states distinguish one from another by the sign of the component p_0 of the canonical 4-momentum p_k. But energy $E=|p_0|$ is positive for both states: particle and antiparticle. Hamiltonian H defined as a quantity canonically conjugate to time x^0 does not coincide with energy E defined as integral of component T^{00} of the energy-momentum. They may coincide (more exactly $E=-H$) only in the case, when there is no pair production. Generally speaking, E and H are different quantities already in relativistic classical mechanics [15].

In the conventional second quantization [11, 12, 13, 14] the particle and the antiparticle are considered as independent dynamical objects. In this case energy E and Hamiltonian H coincide, vacuum state is nonstationary for nonlinear equation and operator ψ contains both creation operators and annihilation operators. In this case a solution of the scattering problem is possible only by means of perturbative methods.

In the correct statement of the second quantization problem [16] the emlon is considered as a main object of dynamics. Particle and antiparticle are considered as different states of the emlon. In this case the energy E and Hamiltonian H are different quantities. Operator ψ contains only annihilation operators and operator ψ^* contains only creation operators. In this case the vacuum state is stationary for nonlinear equation, and the scattering problem can be solved exactly without a use of perturbative methods. In this case it appears that pairs are not created in the case of polynomial form of nonlinear term. It is natural, because being incorporated into wave function, the κ-field is not taken into account.

But why do the pair production appear in the conventional case? In the conventional method of the second quantization the main dynamical object: emlon is divided into parts: particles and antiparticles. After solution of the dynamics problem one needs to unite the parts of the whole object: emlon. However, because of perturbative methods of solution the exact unification fails. The remainders of this approximate unification form the produced pairs. In general, the conventional approach to the second quantization is not consecutive. Using inconsecutive methods of investigation and some ingenuity, one can obtain any result which one wishes.

The action (6.1) - (6.5) contains time derivatives of the κ-field. It means that the κ-field form a dynamic system which can exist without a source and can escape from its source. This example shows, that such a simple logical procedure as the logical reloading changes approach to investigation of elementary particle dynamics. It appears that a simple pointlike particle generates the force field κ which is responsible for pair production and for a change of the effective particle mass. Being an abstract construction, the quantum theory, does not determine such an element of the elementary particle arrangement as the κ-field.

At quantum approach to theory of elementary particles there is a necessity of unification of quantum principles with principles of the relativity theory. Unification of these principles leads to the quantum field theory. The logical reloading in the particle dynamics removes this necessity, because dynamics of stochastic relativistic particles describes automatically all properties of elementary particles, including the problem of pair production.

7 The particle described by the Dirac equation

The Dirac particle (fermion) described by the Dirac equation has no internal structure, if it is described in the framework of quantum theory. At the conventional approach the fermion is a pointlike particle, which mass, spin, charge and magnetic moment are quantum numbers which are ascribed to the Dirac particle. After the logical reloading in the particle dynamics the Dirac equation is considered as a dynamic equation for a statistical ensemble of stochastic particles [17, 18, 19, 20].

After the proper change of variables [19] the action for the Dirac particle $\mathcal{S}_{\mathrm{D}}$ takes the form

$$\mathcal{S}_{\mathrm{D}}: \qquad \mathcal{A}_D[j,\varphi,\kappa,\boldsymbol{\xi}] = \int \mathcal{L} d^4x, \qquad \mathcal{L} = \mathcal{L}_{\mathrm{cl}} + \mathcal{L}_{\mathrm{q1}} + \mathcal{L}_{\mathrm{q2}} \tag{7.1}$$

$$\mathcal{L}_{\mathrm{cl}} = -m\rho - \hbar j^i\partial_i\varphi - \frac{\hbar j^l}{2\left(1+\boldsymbol{\xi}\mathbf{z}\right)}\varepsilon_{\alpha\beta\gamma}\xi^\alpha\partial_l\xi^\beta z^\gamma, \qquad \rho \equiv \sqrt{j^l j_l} \tag{7.2}$$

$$\mathcal{L}_{\mathrm{q1}} = 2m\rho\sin^2(\frac{\kappa}{2}) - \frac{\hbar}{2}S^l\partial_l\kappa, \tag{7.3}$$

$$\mathcal{L}_{\mathrm{q2}} = \frac{\hbar(\rho+j_0)}{2}\varepsilon_{\alpha\beta\gamma}\partial^\alpha\frac{j^\beta}{(j^0+\rho)}\xi^\gamma - \frac{\hbar}{2(\rho+j_0)}\varepsilon_{\alpha\beta\gamma}\left(\partial^0 j^\beta\right)j^\alpha\xi^\gamma \tag{7.4}$$

Lagrangian is a function of 4-vector j^l, scalar φ, pseudoscalar κ, and unit 3-pseudovector $\boldsymbol{\xi}$, which is connected with the spin 4-pseudovector S^l by means of the relations

$$\xi^\alpha = \rho^{-1}\left[S^\alpha - \frac{j^\alpha S^0}{(j^0+\rho)}\right], \qquad \alpha = 1,2,3; \qquad \rho \equiv \sqrt{j^l j_l} \tag{7.5}$$

$$S^0 = \mathbf{j}\boldsymbol{\xi}, \qquad S^\alpha = \rho\xi^\alpha + \frac{(\mathbf{j}\boldsymbol{\xi})j^\alpha}{\rho+j^0}, \qquad \alpha = 1,2,3 \tag{7.6}$$

Let us produce dynamical disquntization, when all derivatives ∂_l are projected on the direction of the 4-current j^k

$$\partial_l \rightarrow \frac{j_l j^k}{j_s j^s}\partial_k \tag{7.7}$$

As a result of the dynamical disquantization the statistical ensemble $\mathcal{S}_{\mathrm{D}}$ of stochastic Dirac particles $\mathcal{S}_{\mathrm{Dst}}$ turns to the statistical ensemble $\mathcal{S}_{\mathrm{Dqu}}$ of deterministic particles $\mathcal{S}_{\mathrm{Dcl}}$. The action for $\mathcal{S}_{\mathrm{Dqu}}$ has the form

$$\mathcal{A}_{\mathrm{Dqu}}[x,\boldsymbol{\xi}] = \int \mathcal{A}_{\mathrm{Dcl}}[x,\boldsymbol{\xi}]d\boldsymbol{\tau}, \qquad \mathbf{d}\boldsymbol{\tau} = d\tau_1 d\tau_2 d\tau_3 \tag{7.8}$$

where the action for $\mathcal{S}_{\mathrm{Dcl}}$ has the form [19]

$$\mathcal{S}_{\mathrm{Dcl}}: \qquad \mathcal{A}_{\mathrm{Dcl}}[x,\boldsymbol{\xi}] = \int\left\{-\kappa_0 m\sqrt{\dot{x}^i\dot{x}_i} + \hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})} + \hbar\frac{(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}}{2\sqrt{\dot{x}^s\dot{x}_s}(\sqrt{\dot{x}^s\dot{x}_s}+\dot{x}^0)}\right\}d\tau_0 \tag{7.9}$$

where $\dot{\mathbf{x}} \equiv d\mathbf{x}/d\tau_0$ and $\mathbf{z}$ is a constant 3-vector, $\kappa_0 = \pm 1$. The deterministic dynamic system $\mathcal{S}_{\mathrm{Dcl}}$ has 10 degrees of freedom. Six translational degrees of freedom are described by variables $\mathbf{x}$ and four rotational degrees of freedom are described by variables $\boldsymbol{\xi}$. Rotational degrees of freedom are described nonrelativistically, although all operations leading from the initial action to the relation (7.8), are relativistically covariant, including (7.7). World line of $\mathcal{S}_{\mathrm{Dcl}}$ is a helix with timelike axis.

Helical shape of the Dirac particle world line explains freely existence of the particle spin and of the magnetic moment by means of the rotation along the helix. The quantum approach cannot explain such an arrangement of the Dirac particle. Thus, arising after logical reloading in the particle dynamics, the statistical approach admits one to investigate arrangement of elementary particles. It is surprising, how the logical reloading produces essential changes in the existing theory, permuting only some fundamental statements of a theory.

8 Mobility of the boundary between the particle dynamics and the space-time geometry

The particle motion occurs in the space-time, and properties of the space-time are essential for description of the particle motion. The boundary between the properties of the space-time and properties of laws of motion (dynamics) is indefinite. One may choose simple properties of the space-time geometry and obtain complicated laws of dynamics. On the contrary, one may choose a simple dynamics (free particle motion) and obtain a complicated space-time geometry. It is possible intermediate version, when dynamics and space-time geometry are not very simple. Historically the boundary between physics and space-time geometry moved towards space-time geometry. This process may be qualified as the physics geometrization. One can see several steps of the physics geometrization: (1) conservation laws as a corollaries of the space-time geometry symmetry, (2) special relativity, (3) general relativity, (4) five-dimensional geometry of Kaluza-Klein, where motion of a charged particle in the given electromagnetic and gravitational fields is described as a free particle motion in the Kaluza-Klein space-time geometry [21]. In the twentieth century the physics geometrization stopped, because we knew a small part of possible space-time geometries. The Riemannian geometries were considered as the most general kind of space-time geometries. In reality, the set of Riemannian geometries is a small part of possible space-time geometries. The most general space-time geometry is described completely in terms of the world function and only in terms of world function. Such a geometry is called the physical geometry [22].

In the classical physics, where gravitational field and electromagnetic field are the only possible force fields, the Kaluza-Klein representation realizes the complete physics geometrization. But this geometrization is not complete in microcosm, where the quantum effects are essential. Besides, the Riemannian geometry which is used in the Kaluza-Klein description is rather complicated. The Riemannian geometry is founded on several basic concepts: (1) concepts of topology, (2) concepts of local geometry such as dimension, coordinate system, metric tensor and parallel transport. A work with concepts of the Riemannian geometry is not simpler, than the work with numerous concepts of dynamics. As a result one prefers to work with customary concepts of dynamics.

9 Logical reloading in space-time geometry

If one wants to realize the physics geometrization in microcosm, one needs to use all possible space-time geometries, including discrete space-time geometries. Any space-time geometry is obtained as some generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. Conventionally describing $\mathcal{G}_{\mathrm{E}}$, one uses representation, where basic concepts are (1) topological concepts and (2) concepts of local geometry. Such a representation of $\mathcal{G}_{\mathrm{E}}$ will be referred to as vector representation (or V-representation), because it uses essentially the linear vector space. The Riemannian geometry and the proper

Euclidean geometry are continuous geometries. They have only one geometrical quantity which is common with the discrete space-time geometry. This quantity is a distance. If one wants to obtain the discrete space-time geometry as a generalization of $\mathcal{G}_{\mathrm{E}}$, one needs to present $\mathcal{G}_{\mathrm{E}}$ in terms of distance ρ_{E} and only in terms of distance. It is possible [22]. Such a representation of $\mathcal{G}_{\mathrm{E}}$ is a result of the logical reloading, when all basic concepts of V-representation: concepts of topology and concepts of local geometry are expressed via the distance ρ_{E} and only via the distance ρ_{E}. Instead of distance one may use the world function $\sigma_{\mathrm{E}} = \frac{1}{2}\rho_{\mathrm{E}}^2$, because the world function is always real for the space-time geometry. After such a logical reloading the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ turns to a monistic conception, where all geometric quantities and concepts are expressed via the only basic quantity: world function σ_{E}. Such a representation of $\mathcal{G}_{\mathrm{E}}$ is called σ-representation of $\mathcal{G}_{\mathrm{E}}$. Replacing σ_{E} by the world function σ of the space-time geometry $\mathcal{G}$ in all definitions of $\mathcal{G}_{\mathrm{E}}$, the space-time geometry $\mathcal{G}$ is obtained from $\mathcal{G}_{\mathrm{E}}$. The geometry which is completely described by the world function will be referred to as physical geometry. Such a procedure of replacement of σ_{E} by σ may be interpreted as a deformation of the proper Euclidean geometry [23, 24].

For instance, in $\mathcal{G}_{\mathrm{E}}$ the segment $\mathcal{T}_{[PQ]}$ of the straight line between the points P and Q in $\mathcal{G}_{\mathrm{E}}$ is defined as a set of points R, satisfying the equation

$$\mathcal{T}_{[PQ]} = \left\{ R | \sqrt{2\sigma_{\mathrm{E}}(P,R)} + \sqrt{2\sigma_{\mathrm{E}}(R,Q)} = \sqrt{2\sigma_{\mathrm{E}}(P,Q)} \right\}, \quad \rho_{\mathrm{E}} = \sqrt{2\sigma_{\mathrm{E}}} \tag{9.1}$$

In the space-time geometry $\mathcal{G}$ the straight line segment is defined by the same relation

$$\mathcal{T}_{[PQ]} = \left\{ R | \sqrt{2\sigma(P,R)} + \sqrt{2\sigma(R,Q)} = \sqrt{2\sigma(P,Q)} \right\} \tag{9.2}$$

but the world function σ distinguishes from σ_{E}. Generally speaking, the segment $\mathcal{T}_{[PQ]}$ in $\mathcal{G} = \{\sigma, \Omega\}$ is not a one-dimensional set of points, whereas $\mathcal{T}_{[PQ]}$ in $\mathcal{G}_{\mathrm{E}} = \{\sigma_{\mathrm{E}}, \Omega\}$ is a one-dimensional set of points. Mathematically it means that in $\mathcal{G}_{\mathrm{E}} = \{\sigma_{\mathrm{E}}, \Omega\}$ any section $\mathcal{S}\left(S, \mathcal{T}_{[PQ]}\right)$ of $\mathcal{T}_{[PQ]}$ at a point $S \in \mathcal{T}_{[PQ]}$ consists of the only point S,

$$\mathcal{S}\left(S, \mathcal{T}_{[PQ]}\right) = \{R | \sigma_{\mathrm{E}}(P,R) = \sigma_{\mathrm{E}}(P,S) \wedge \sigma_{\mathrm{E}}(Q,R) = \sigma_{\mathrm{E}}(Q,S)\} = \{S\}, \quad S \in \mathcal{T}_{[PQ]} \tag{9.3}$$

whereas in $\mathcal{G} = \{\sigma, \Omega\}$ the same section of $\mathcal{T}_{[PQ]}$ at a point $S \in \mathcal{T}_{[PQ]}$ consists, generally speaking, of many points

$$\mathcal{S}\left(S, \mathcal{T}_{[PQ]}\right) = \{R | \sigma(P,R) = \sigma(P,S) \wedge \sigma(Q,R) = \sigma(Q,S)\} \subset \mathcal{T}_{[PQ]}, \quad S \in \mathcal{T}_{[PQ]} \tag{9.4}$$

Here $\mathcal{S}\left(S, \mathcal{T}_{[PQ]}\right)$ is a section of the segment $\mathcal{T}_{[PQ]}$ at the point S. On one hand, one equation (9.2) in n-dimensional space describes, generally speaking, $(n-1)$-dimensional surface, and it is natural. On the other hand, the segment (9.2) is a segment of a straight line in the space-time geometry $\mathcal{G}$, and it seems rather strange, why this segment is not one-dimensional. The segment $\mathcal{T}_{[PQ]}$ is one-dimensional in $\mathcal{G}_{\mathrm{E}} = \{\sigma_{\mathrm{E}}, \Omega\}$. It seems that $\mathcal{T}_{[PQ]}$ is to be one-dimensional in any space-time geometry. But such an approach is possible only, if the straight line segment $\mathcal{T}_{[PQ]}$ in $\mathcal{G}_{\rm E}$ is considered as a basic object of the space-time geometry. However, if the world function is the basic object of space-time geometry, the straight line segment $\mathcal{T}_{[PQ]}$ in $\mathcal{G}_{\rm E}$ is a derivative object, which should be defined by the relation (9.1). In general, the straight line segment $\mathcal{T}_{[PQ]}$ is not one-dimensional. But why is it one-dimensional in $\mathcal{G}_{\rm E} = \{\sigma_{\rm E}, \Omega\}$? It is one-dimensional in $\mathcal{G}_{\rm E}$, because of special properties of the world function $\sigma_{\rm E}$. The proper Euclidean geometry $\mathcal{G}_{\rm E}$ is a degenerate geometry, and different geometrical objects of arbitrary space-time geometry, may coincide in $\mathcal{G}_{\rm E}$.

We consider this effect in the example of a circular cylinder. In $\mathcal{G}_{\rm E}$ it is defined by its axis and a point P on its surface. Let F_1, F_2 be two points on the axis of the circular cylinder. The cylinder $Cl_{PF_1F_2}$ is defined as a set of points R

$$Cl_{PF_1F_2} = \{R | S_{F_1F_2R} = S_{F_1F_2P}\} \tag{9.5}$$

where $S_{F_1F_2R}$ is the area of the triangle with vertices at the points F_1, F_2, R. The area is calculated by means of the Heron's formula via side lengths of the triangle. The areas $S_{F_1F_2R}$ and $S_{F_1F_2P}$ are expressed via world functions of corresponding points. Let $\mathcal{T}_{[F_1F_2]}$ be the straight line segment between points F_1, F_2 and the point $F_3 \in \mathcal{T}_{[F_1F_2]}$. Let $F_3 \neq F_1$, then in $\mathcal{G}_{\rm E}$ the shape of the circular cylinder depends only on the axis $\mathcal{T}_{[F_1F_2]}$, but not on a choice of points on this axis, and

$$Cl_{PF_1F_2} = Cl_{PF_1F_3}, \qquad F_3 \in \mathcal{T}_{[F_1F_2]} \tag{9.6}$$

However, in the arbitrary space-time geometry $\mathcal{G} = \{\sigma, \Omega\}$, generally speaking, $Cl_{PF_1F_2} \neq Cl_{PF_1F_3}$, and in $\mathcal{G} = \{\sigma, \Omega\}$ there are many cylinders, corresponding to one circular cylinder in the proper Euclidean geometry. From viewpoint of V-representation it is interpreted as a splitting of the Euclidean cylinder in $\mathcal{G} = \{\sigma, \Omega\}$. From viewpoint of σ-representation the fact, that shape of cylinders $Cl_{PF_1F_2}$ and $Cl_{PF_1F_3}$ are different, in general, is natural. From this viewpoint the equation (9.6) means a degeneration of cylinders in the Euclidean geometry. Interpretation of (9.6) as a degeneration is a more correct geometrical interpretation, because it does not use such an auxiliary structure as the linear vector space.

Unfortunately, the degenerate character of $\mathcal{G}_{\rm E}$ is hardly perceived by mathematicians. For instance, Blumental constructed the distance geometry [25], where the distance was the basic quantity, as in physical geometry. Unfortunately, he does not use the deformation principle. He consider a curve (and, in particular, the straight line) as a one-dimensional set of points. He was forced to define a curve as a continuous mapping of the numerical interval $(0, 1)$ onto the point set of the distance geometry. As a result his distance geometry appeared to be inconsecutive in sense that the distance geometry contains basic concepts which contain not only concept of distance.

At description of a geometry $\mathcal{G}_{\rm E}$ the σ-representation may be called as the metric approach to geometry. At the metric approach a construction of geometrical objects in $\mathcal{G}_{\rm E}$ does not refer to dimension of $\mathcal{G}_{\rm E}$ or to a coordinate system. At the metric approach the dimension of $\mathcal{G}_{\mathrm{E}}$ and the coordinate system are not basic objects of $\mathcal{G}_{\mathrm{E}}$. They are defined in terms of the world function σ_{E} of $\mathcal{G}_{\mathrm{E}}$ and only in terms of σ_{E}. The definition (9.1) of a straight line segment $\mathcal{T}_{[PQ]}$ is an example of a geometrical object definition in terms of the world function. Construction of geometric objects in terms of only world function is necessary to identify the same physical body in different space-time geometries.

Physical space-time geometries are multivariant, generally speaking. Multivariance of a geometry means that at the point C there are many vectors $\mathbf{CD}$, $\mathbf{CD}'$, $\mathbf{CD}''$,... which are equivalent to the vector $\mathbf{AB}$ at the point A, but vectors $\mathbf{CD}$, $\mathbf{CD}'$, $\mathbf{CD}''$,... are not equivalent between themselves. Such a situation takes place in physical geometries. It is connected with the intransitivity of the equivalence relation. In the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ the equivalence relation is defined as follows. Vector $\mathbf{CD}$ is equivalent to vector $\mathbf{AB}$ ($\mathbf{CD}\mathrm{eqv}\mathbf{AB}$)

$$(\mathbf{CD}\mathrm{eqv}\mathbf{AB}): \quad (\mathbf{CD}.\mathbf{AB}) = |\mathbf{CD}|\cdot|\mathbf{AB}| \wedge |\mathbf{CD}| = |\mathbf{AB}| \tag{9.7}$$

$$(\mathbf{CD}.\mathbf{AB}) = \sigma(C,B) + \sigma(D,A) - \sigma(C,A) - \sigma(D,B) \tag{9.8}$$

$$|\mathbf{AB}| = \sqrt{2\sigma(A,B)} \tag{9.9}$$

where $\sigma = \sigma_{\mathrm{E}}$. Of course, in any physical geometry the equivalency of two vectors is defined by equations (9.7) – (9.9) also. In $\mathcal{G}_{\mathrm{E}}$ at the point C there is one and only one vector $\mathbf{CD}$ which is equivalent to vector $\mathbf{AB}$. But in arbitrary physical geometry $\mathcal{G}$ there may be many vectors $\mathbf{CD}$, $\mathbf{CD}'$, $\mathbf{CD}''$,... which are equivalent to the vector $\mathbf{AB}$. The vectors $\mathbf{CD}$, $\mathbf{CD}'$, $\mathbf{CD}''$,.. may be not equivalent between themselves. It means that the equivalence relation is intransitive in $\mathcal{G}$ and the geometry $\mathcal{G}$ is not axiomatizable, because in any axiomatizable geometry the equivalence relation is transitive.

Most mathematicians dislike the physical geometries, because they are nonaxiomatizable, and in physical geometries there are no theorems which are used at the construction of the proper Euclidean geometry. Some of mathematicians state even that nonaxiomatizable geometries do not exist. Such an approach is connected with the fact that the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ was the only geometry, which has been studied during two thousand years. The study of $\mathcal{G}_{\mathrm{E}}$ consists of proof of numerous theorems. As a result many scientists believe that these theorems form a content of $\mathcal{G}_{\mathrm{E}}$. They cannot imagine Euclidean geometry without theorems. In reality, theorems are attributes of the *Euclidean geometry construction*, but not attributes of the Euclidean geometry itself. Content of the Euclidean geometry consists of a set of the geometry statements $\mathcal{P}_{\mathrm{E}}$. In the physical geometry $\mathcal{G}$ the geometrical statements $\mathcal{P}$ are obtained from the Euclidean statements $\mathcal{P}_{\mathrm{E}}$ by means of deformation, and there is no necessity to prove any theorems.

However, it is a right of mathematicians, when they do not to consider nonaxiomatizable geometries, because they have a right to study only part of possible space-time geometries. But physicists have not such a possibility. If the space-time geometry depends on the matter distribution in the space-time, then investigating the space-time geometry, physicists are to consider all possible (physical) space-time geometries. They have no right to say: "We shall consider only Riemannian space-time geometries, because only they has been investigated properly." Consideration of all possible space-time geometries (but not only Riemannian ones) in the general relativity theory leads to the extended general relativity, where the dark holes absent because of induced antigravitation [1, 2].

Note that even the space-time geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$ is multivariant with respect to spacelike vectors. For instance, in $\mathcal{G}_{\mathrm{M}}$ vectors with coordinates $\{r_1, r_1 \cos\phi_1, r_1 \sin\phi_1, z\}$ and $\{r_2, r_2 \cos\phi_2, r_2 \sin\phi_2, z\}$ are equivalent to spacelike vector $(0,0,0,z)$ at arbitrary values of $r_{1,}r_2, \phi_1, \phi_2$, but they are not equivalent between themselves, generally speaking. As a result the particles having spacelike world line (tachyons) may exist. But world line of a tachyon wobbles with infinite amplitude, and a single tachyon cannot be detected (but it does exist). The tachyon gas may be detected by its gravitational field. The tachyon gas is the best candidate for the dark matter [26]. Of course, this fact does not exclude existence of other particle of the dark matter. However, existence of only tachyon gas explains freely the phenomenon of the dark matter.

Discrete space-time geometry is multivariant. This multivariance generates wobbling of world lines of elementary particles, which means stochasticity of the elementary particles. Besides, the discrete space-time geometry is formulated in the coordinateless form. Mathematicians cannot construct the Riemannian geometry in the coordinateless form.

10 Inadequacy of the linear vector space operations in multivariant geometry

Geometrical vector (g-vector) $\mathbf{AB}$ is defined as a the ordered set $\mathbf{AB} = \{A, B\}$ of two points $A, B \in \Omega$. Here Ω is the set of points (events) of the space-time, where the geometry is given. We use the term g-vector (vector), because there are linear vectors (linvectors) u, which are defined as elements of the linear vector space $\mathcal{L}_n$. Linvectors $u \in \mathcal{L}_n$ are abstract quantities, whose properties are defined by a system of axioms. In particular, operations of summation of linvectors and multiplication of a linvector by a real number are defined in $\mathcal{L}_n$. Under some conditions the operations on linvectors may be applied to g-vectors.

Linvectors and g-vectors have different properties. Any linvector exists in one copy, whereas there are many g-vectors $\mathbf{CD}$ which are equivalent to the g-vector $\mathbf{AB}$. Geometric vector $\mathbf{CD}$ is equivalent (equal) to g-vector $\mathbf{AB}$ ($\mathbf{CD}\mathrm{eqv}\mathbf{AB}$), if the conditions (9.7) – (9.9) are satisfied. Definition (9.7) – (9.9) of two g-vectors equivalence depends only on the world function. It does depend neither on dimension, nor on the coordinate system. Definition (9.7) – (9.9) of two g-vectors equivalence can be used in any physical geometry.

Let $S_{\mathbf{AB}}$ be a set of g-vectors $\mathbf{CD}$, which are equivalent to g-vector $\mathbf{AB}$. If the equivalence relation is transitive, the set $S_{\mathbf{AB}}$ is a equivalence class $[\mathbf{AB}]$ of the g-vector $\mathbf{AB}$. It contains only g-vectors which are equivalent between themselves.

In this case any equivalence class $[\mathbf{AB}]$ may be corresponded by some linvector $u \in \mathcal{L}_n$, and this correspondence will be one-to-one, because any equivalence class exist only in one copy. If the equivalence relation is intransitive and the set $S_{\mathbf{AB}}$ does not form an equivalence class, the correspondence between the linvectors and g-vectors cannot be established, because the set $S_{\mathbf{AB}}$ contains g-vectors which are not equivalent. As a result the operations of the linear vector space $\mathcal{L}_n$ are not adequate in the multivariant geometry, where the equivalence relation is intransitive.

Formally one may introduce summation of g-vectors in multivariant geometry, but this summation will be many-valued. One may summarize vectors $\mathbf{AB}$ and $\mathbf{CD}$, if $B = C$.

$$\mathbf{AB} + \mathbf{BD} = \mathbf{AD} \tag{10.1}$$

However, let one needs to sum g-vectors $\mathbf{AB}$ and $\mathbf{CD}$, and $B \neq C$. Let g-vector $\mathbf{PQ} = \mathbf{AB}+\mathbf{CD}$, where the point P is given, and the point Q should be determined. One obtains

$$\mathbf{PQ} = \mathbf{PF} + \mathbf{FQ} \tag{10.2}$$

where points F and Q are determined from the relations

$$(\mathbf{PF}\mathrm{eqv}\mathbf{AB}) \wedge (\mathbf{FQ}\mathrm{eqv}\mathbf{CD}) \tag{10.3}$$

In the multivariant geometry the equations (10.3) have many solutions for the points F and Q, and the operation of summation appears to be many-valued. In the single-variant geometry the relations (10.3) have unique solution for points F and Q and the summation (10.2) is defined one-to-one. Multiplication of g-vector by a number and decomposition of g-vector appear also many-valued in the multivariant geometry

11 σ-representation of the proper Euclidean geometry

In the σ-representation of $\mathcal{G}_{\mathrm{E}}$ there are two kinds of relations: (1) general geometric relations and (2) special relations. The general geometric relations are the relations, which are written only in terms of the world function. The general geometric relations are valid for any physical geometry. The general geometric relations describe properties of the linear vector space without a reference to it.

The first general geometric relation is the definition of the scalar product of two g-vectors (9.8). Definition of the two g-vector equivalence (9.7) – (9.9) is also a general geometric relation.

Linear dependence of n g-vectors $\mathbf{P}_0\mathbf{P}_1, \mathbf{P}_0\mathbf{P}_2, ...\mathbf{P}_0\mathbf{P}_n$ is defined by the relation,

$$F_n(\mathcal{P}_n) = 0, \quad F_n(\mathcal{P}_n) \equiv \det||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)||, \qquad i,k = 1,2,...n \tag{11.1}$$

where $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$ and $F_n(\mathcal{P}_n)$ is the Gram's determinant. Vanishing of the Gram's determinant is the necessary and sufficient condition of the linear dependence of n g-vectors. Condition of linear dependence relates usually to the properties of the linear vector space. It seems rather meaningless to use it, if the linear vector space cannot be introduced. Nevertheless, the relation (11.1) written as a general geometric relation describes some general geometric properties of g-vectors, which in the proper Euclidean geometry transform to the property of linear dependence. In particular, the dimension of the proper Euclidean geometry is defined in terms of the world function by means of the relations of the type (11.1) as a maximal number of linear independent g-vectors, which is possible in the Euclidean space. This circumstance seems to be rather unexpected, because in conventional representation (vector representation [27]) of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ the geometry dimension is postulated in the beginning of the geometry construction.

For instance, a construction of the Riemannian geometry begins conventionally from definition of a manifold, its dimension and coordinate system on the manifold. It means that coordinate system is considered as a basic object of the Riemannian geometry, although the coordinate system is only a means of description which may be changed in different manners.

12 Specific properties of the n-dimensional Euclidean space

Along of general geometric properties, connecting mainly with the properties of the linear vector space, there are special geometric relations, describing properties of the world function. For instance, there are relations, which are necessary and sufficient conditions of the fact, that the world function σ_{E} is the world function of n-dimensional Euclidean space. They have the form [22]:

I. Definition of the dimension:

$$\exists\mathcal{P}^n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \qquad F_n(\mathcal{P}^n) \neq 0, \qquad F_k\left(\Omega^{k+1}\right) = 0, \qquad k > n \tag{12.1}$$

where $F_n(\mathcal{P}^n)$ is the n-th order Gram's determinant (11.1). Geometric vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$ are basic g-vectors of the rectilinear coordinate system K_n with the origin at the point P_0. The metric tensors $g_{ik}(\mathcal{P}^n)$, $g^{ik}(\mathcal{P}^n)$, $i, k = 1, 2, ...n$ in K_n are defined by the relations

$$\sum_{k=1}^{k=n} g^{ik}(\mathcal{P}^n)\, g_{lk}(\mathcal{P}^n) = \delta_l^i, \qquad g_{il}(\mathcal{P}^n) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_l), \qquad i, l = 1, 2, ...n \tag{12.2}$$

$$F_n(\mathcal{P}^n) = \det ||g_{ik}(\mathcal{P}^n)|| \neq 0, \qquad i, k = 1, 2, ...n \tag{12.3}$$

II. Linear structure of the Euclidean space:

$$\sigma_{\mathrm{E}}(P, Q) = \frac{1}{2}\sum_{i,k=1}^{i,k=n} g^{ik}(\mathcal{P}^n)\,(x_i(P) - x_i(Q))\,(x_k(P) - x_k(Q)), \qquad \forall P, Q \in \Omega \tag{12.4}$$

where coordinates $x_i(P)$, $x_i(Q)$, $i=1,2,...n$ of the points P and Q are covariant coordinates of the g-vectors $\mathbf{P}_0\mathbf{P}$, $\mathbf{P}_0\mathbf{Q}$ respectively in the coordinate system K_n. The covariant coordinates are defined by the relation

$$x_i(P) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}), \qquad i=1,2,...n \tag{12.5}$$

III: The metric tensor matrix $g_{lk}(\mathcal{P}^n)$ has only positive eigenvalues g_k

$$g_k > 0, \qquad k=1,2,...,n \tag{12.6}$$

IV. The continuity condition: the system of equations

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}) = y_i \in \mathbb{R}, \qquad i=1,2,...n \tag{12.7}$$

considered to be equations for determination of the point P as a function of coordinates $y=\{y_i\}$, $i=1,2,...n$ has always one and only one solution. Conditions I – IV contain a reference to the dimension n of the Euclidean space, which is defined by the relations (12.1).

All relations I – IV are written in terms of the world function. They are constraints on the form of the world function of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. Constraints (12.1), determining the dimension via the form of the world function, look rather unexpected. They contain a lot of constraints imposed on the world function of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, and they are necessary. At the conventional approach to geometry one uses a very simple supposition: "Let the dimension of the Euclidean space be n." instead of numerous constraints (12.1).

At the vector representation of the proper Euclidean geometry, which is based on a use of the linear vector space, the dimension is considered as a primordial property of the linear vector space and as a primordial property of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. Situation, when the geometry dimension is different at different points of the set Ω, or when it is indefinite, is not considered. At the vector representation of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ one does not distinguish between the general geometric relations and the specific relations of the geometry.

At the metric approach to geometry, when the space-time geometry is described in terms of only distance ρ or in terms of only world function $\sigma=\rho^2/2$, any modification of the space-time geometry looks very simple. To obtain a modification of a geometry, one replaces world function and obtains a modified geometry described by the new world function. If the geometry is described by means of several fundamental concepts, any modification of the geometry needs a modification of all fundamental concepts. This modification of different fundamental concepts is to be concerted, in order the modified geometry be consistent. The more number of the basic concepts the difficult agreement between the modified concepts. The monistic conception of a geometry, when there is only one fundamental quantity is the best conception, because the problem of agreement of different basic modified concepts is absent. From this viewpoint the metric approach to the space-time geometry is the best approach. It gives the most general description of the space-time geometry.

Thus, logical reloading in $\mathcal{G}_{\mathrm{E}}$ admits one to construct maximal number of different space-time geometries without any problems.

13 Skeleton conception of elementary particle dynamics

At the quantum approach one considers only pointlike particles, and it is useless to discuss the particle structure. In the framework of this approach there exist composite particles consisting of several pointlike particles. For instance, proton and other hadrons consist of quarks connected by gluons. Quarks are never directly observed or found in isolation; they can be found only within hadrons. In other words, quarks are observed as elements of the proton (and hadron) structure, which cannot be extracted from proton and from other hadrons. For this reason it would be more natural to consider quarks as elements of the hadron structure. Unfortunately, the quantum theory cannot consider structure of elementary particles. It can consider only pointlike particles or aggregations of pointlike particles. Mathematical formalism of quantum field theory does enable to describe composite elementary particles. It can describe only aggregations of pointlike particles. Such a property of quantum theory is conditioned by the fact that the space-time is continuous in the quantum field theory, and divisibility of geometrical objects is unrestricted.

In the discrete space-time geometry, where there is a minimal elementary length, the divisibility of geometrical objects is restricted. In such a situation a possible structure of geometrical objects seems to be very natural. Statistical approach to analysis of the Dirac equation shows [17, 19, 20], that the world line associated with the Dirac particle is a helix with timelike axis. Rotational degrees of freedom are described nonrelativistically and one cannot decide distinctly, whether the rotation happens with superluminal velocity or not. The Dirac particle can be described as a rotator. In the discrete space-time geometry the helix can be obtained, if the Dirac particle is a composite one and its skeleton consists of more than two points [28]. In this case it is reasonable to speak about the Dirac particle structure.

Any geometrical object $g_{\mathcal{P}_n,\sigma}$ in the space-time is described by its skeleton $\mathcal{P}_n$ and envelope of the skeleton. A skeleton $\mathcal{P}_n$ is a set of $n+1$ space-time points

$$\mathcal{P}_n = \{P_0, P_1, ...P_n\}$$

connected rigidly between themselves. It means that the distances

$$\mu_{ik} = \sqrt{2\sigma\left(P_i, P_k\right)}, \quad i, k = 0, 1, ..n \tag{13.1}$$

are not changed at any displacement of the geometric object $g_{\mathcal{P}_n,\sigma}$. The envelope of the skeleton is described as a set of points R, which are zeros of some function of distances between the points $\{\mathcal{P}_n, R\}$. See details in [29]. Examples of geometrical objects are the segment of straight line $\mathcal{T}_{[PQ]}$ (9.2) and the circular cylinder $Cl_{PF_1F_2}$, defined by (9.5).

It is supposed that any physical body has a shape of a geometrical object. Not any subset of points of the space-time is a geometrical object. Tracing the motion of a skeleton $\mathcal{P}_n$ of a physical body (elementary particle), one can trace the motion of a physical body. The distances (13.1) contain all geometric information on the position of a geometrical object except for its orientation. The geometrical object $g_{\mathcal{P}_n,\sigma}$ orientation is described by its skeleton $\mathcal{P}_n$. Geometrical vector $\mathbf{P}_0\mathbf{P}_1$ is the leading vector, which determines the physical body motion in the space-time. The length $|\mathbf{P}_0\mathbf{P}_1|$ of the leading g-vector determines the link length of the world chain $\mathcal{C}$, which describes motion of the particle skeleton $\mathcal{P}_n$ in the space-time.

The world chain $\mathcal{C}$ of connected skeletons is defined by the relation

$$\mathcal{C} = \bigcup_{s=-\infty}^{s=+\infty} \mathcal{P}_n^{(s)} \tag{13.2}$$

Skeletons $\mathcal{P}_n^{(s)}$ of the world chain are connected in the sense, that the point P_1 of a skeleton is the point P_0 of the adjacent skeleton. It means

$$P_1^{(s)} = P_0^{(s+1)}, \qquad s = ...0, 1, ... \tag{13.3}$$

The geometric vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ is the leading g-vector, which determines the direction of the world chain.

If the particle motion is free, the adjacent skeletons are equivalent

$$\mathcal{P}_n^{(s)}\text{eqv}\mathcal{P}_n^{(s+1)}: \qquad \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\text{eqv}\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}, \qquad i,k = 0,1,...n, \qquad s = ..0,1,.. \tag{13.4}$$

If the particle is described by the skeleton $\mathcal{P}_n^{(s)}$, the world chain (13.4) has $n(n+1)/2$ invariants (13.1).

The main supposition of the skeleton conception of elementary particle dynamics states that the dynamics of elementary particles can be geometrized completely. It means that the motion of any elementary particle can be presented as a free motion in the *true space-time geometry*. Equations (13.4) are written in the form

$$\begin{aligned} &\sigma\left(P_i^{(s+1)}, P_k^{(s)}\right) + \sigma\left(P_k^{(s+1)}, P_i^{(s)}\right) - \sigma\left(P_i^{(s+1)}, P_i^{(s)}\right) - \sigma\left(P_k^{(s+1)}, P_k^{(s)}\right) \\ = \;\; & 2\sigma\left(P_i^{(s)}, P_k^{(s)}\right), \quad i,k = 0,1,...n, \quad s \in \mathbb{N} \end{aligned} \tag{13.5}$$

$$\sigma\left(P_i^{(s+1)}, P_k^{(s+1)}\right) = \sigma\left(P_i^{(s)}, P_k^{(s)}\right), \quad i,k = 0,1,...n, \quad s \in \mathbb{N} \tag{13.6}$$

$$P_0^{(s+1)} = P_1^{(s)}, \quad s \in \mathbb{N} \tag{13.7}$$

Equations (13.7) form D trivial dynamic equations where D is the coordinate dimension of the space-time geometry (the number of coordinates labelling points of the space-time). The number of dynamic variables (coordinates of points $P_1, P_2, ...P_n$) is equal nD. The number of nontrivial dynamic equations (13.5), (13.6) is equal $n\,(n+1)$, which does not coincide with nD, generally speaking. All parameters of a particle can be geometrized and expressed via $n\,(n+1)\,/2$ quantities μ_{ik}, defined by (13.1). In particular, in the case of a pointlike particle where $n = 1$ the only quantity $\mu = \mu_{01}$ is connected with the particle mass m by means of relation

$$m = b\mu \tag{13.8}$$

where b is some universal constant.

The dynamic equations (13.4) describe a free particle motion in true space-time geometry. These dynamic equations may be written in arbitrary space-time geometry. However, then additional force fields appear.

Let us describe equations (13.5) (13.6) in the space time geometry of Minkowski. One obtains

$$\sigma(P,Q) = \sigma_{\mathrm{M}}(P,Q) + d(P,Q) \tag{13.9}$$

where $\sigma_{\mathrm{M}}(P,Q)$ is the world function of the Minkowski space-time geometry. The quantity $d(P,Q)$ determines some force field acting on the particle in the geometry of Minkowski. One obtains instead of (13.5) (13.6)

$$\begin{aligned} &\sigma_{\mathrm{M}}\left(P_i^{(s+1)}, P_k^{(s)}\right) + \sigma_{\mathrm{M}}\left(P_k^{(s+1)}, P_i^{(s)}\right) - \sigma_{\mathrm{M}}\left(P_i^{(s+1)}, P_i^{(s)}\right) - \sigma_{\mathrm{M}}\left(P_k^{(s+1)}, P_k^{(s)}\right) \\ = \ & 2\sigma_{\mathrm{M}}\left(P_i^{(s)}, P_k^{(s)}\right) + 2d\left(P_i^{(s)}, P_k^{(s)}\right) + w\left(P_i^{(s+1)}, P_k^{(s+1)}, P_i^{(s)}, P_k^{(s)}\right), \end{aligned} \tag{13.10}$$

$$i,k = 0,1,...n, \quad s \in \mathbb{N}$$

$$\begin{aligned} &\sigma_{\mathrm{M}}\left(P_i^{(s+1)}, P_k^{(s+1)}\right) - \sigma_{\mathrm{M}}\left(P_i^{(s)}, P_k^{(s)}\right) \\ = \ & -d\left(P_i^{(s+1)}, P_k^{(s+1)}\right) + d\left(P_i^{(s)}, P_k^{(s)}\right), \quad i,k = 0,1,...n, \quad s \in \mathbb{N} \end{aligned} \tag{13.11}$$

where

$$P_0^{(s+1)} = P_1^{(s)}, \quad i,k = 0,1,...n, \quad s \in \mathbb{N} \tag{13.12}$$

$$\begin{aligned} w\left(P_i^{(s+1)}, P_k^{(s+1)}, P_i^{(s)}, P_k^{(s)}\right) &= -d\left(P_i^{(s+1)}, P_k^{(s)}\right) - d\left(P_k^{(s+1)}, P_i^{(s)}\right) + d\left(P_i^{(s+1)}, P_i^{(s)}\right) \\ +d\left(P_k^{(s+1)}, P_k^{(s)}\right), \quad i,k &= 0,1,...n, \quad s \in \mathbb{N} \end{aligned} \tag{13.13}$$

The dynamic equations (13.10) describe motion of the particle (its skeleton) in the force field w.

In the simplest case of pointlike particle, when the skeleton consists of two points the dynamic equations (13.10), (13.11) have the form

$$\left|\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right|_{\mathrm{M}}^2 - \left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{M}}^2 = 2d\left(P_0^{(s)}, P_1^{(s)}\right) - 2d\left(P_1^{(s)}, P_1^{(s+1)}\right) \tag{13.14}$$

$$\begin{aligned} &\left(\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}.\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right)_{\mathrm{M}} - \left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{M}}^2 - 3d\left(P_0^{(s+1)}, P_0^{(s)}\right) \\ = \ & d\left(P_1^{(s+1)}, P_1^{(s)}\right) - d\left(P_0^{(s+1)}, P_1^{(s)}\right) - d\left(P_1^{(s+1)}, P_0^{(s)}\right) \end{aligned} \tag{13.15}$$

where one uses, that $P_1^{(s)} = P_0^{(s+1)}$.

In the case, when the leading vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ is timelike, one can introduce the angle $\phi_{01}^{(s)}$ between the vectors $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ and $\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}$ in the geometry $\mathcal{G}_{\mathrm{M}}$. Let us define

$$\cosh\phi_{01}^{(s)} = \frac{\left(\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}.\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right)_{\mathrm{M}}}{\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{M}}\left|\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right|_{\mathrm{M}}} \tag{13.16}$$

By means of (13.16) one obtains from (13.15)

$$\begin{aligned}
&\cosh\phi_{01}^{(s)} - \frac{\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{M}}}{\left|\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right|_{\mathrm{M}}} \\
= \ &\frac{3d\left(P_0^{(s+1)},P_0^{(s)}\right)+d\left(P_1^{(s+1)},P_1^{(s)}\right)-d\left(P_0^{(s+1)},P_1^{(s)}\right)-d\left(P_1^{(s+1)},P_0^{(s)}\right)}{\left|\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right|_{\mathrm{M}}\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{M}}}
\end{aligned} \tag{13.17}$$

If $|d| \ll \left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{M}}$, then it follows from (13.17)

$$\begin{aligned}
&1+2\sinh^2\frac{\phi_{01}^{(s)}}{2} - 1 - \frac{\left(d\left(P_0^{(s)},P_1^{(s)}\right)-d\left(P_1^{(s)},P_1^{(s+1)}\right)\right)}{\left|\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right|_{\mathrm{M}}^2} \\
= \ &\frac{3d\left(P_0^{(s+1)},P_0^{(s)}\right)+d\left(P_1^{(s+1)},P_1^{(s)}\right)-d\left(P_0^{(s+1)},P_1^{(s)}\right)-d\left(P_1^{(s+1)},P_0^{(s)}\right)}{\left|\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right|_{\mathrm{M}}\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{M}}}
\end{aligned}$$

Then in the geometry $\mathcal{G}_{\mathrm{M}}$ the equation (13.17) has the form

$$\sinh\frac{\phi_{01}^{(s)}}{2} = \frac{\sqrt{4d\left(P_0^{(s+1)},P_0^{(s)}\right)-d\left(P_0^{(s+1)},P_1^{(s)}\right)-d\left(P_1^{(s+1)},P_0^{(s)}\right)}}{\sqrt{2}\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{M}}}+\mathcal{O}\left(\frac{d^2}{\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{M}}^2}\right) \tag{13.18}$$

Thus, if the field $d=0$, the angle between two adjacent links $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ and $\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}$ vanishes. If d is small quantity, then the adjacent link is placed on the cone with the angle ϕ_{01} at the vertex, which is determined by the relation (13.18).

In general, the force field in the rhs of (13.18) depends on three points even in the simplest case of pointlike particle described by two point skeleton. In the limit of continuous geometry it corresponds the case, when dynamic equations contain the vector force field F_k and derivatives $\partial_i F_k$ of F_k. In particular, motion of a particle in the gravitational field described by the Newtonian potential V has the form [30]

$$\begin{aligned}
&\dot{v}_\parallel^2\left(1+\frac{v_\parallel^2}{c^2-2V-\mathbf{v}^2}\right)-2\dot{v}_\parallel\left(|\boldsymbol{\nabla}V|-\frac{(\mathbf{v}\boldsymbol{\nabla}V)\,v_\parallel}{c^2-2V-\mathbf{v}^2}\right)+\left\langle\dot{\mathbf{v}}_\perp^2\right\rangle \\
= \ &-\frac{(\mathbf{v}\boldsymbol{\nabla}V)^2+(\mathbf{v}_\perp\dot{\mathbf{v}}_\perp)^2}{c^2-2V-\mathbf{v}^2}-\frac{1}{c^2}v^\alpha v^\beta\partial_\alpha\partial_\beta V
\end{aligned} \tag{13.19}$$

In the nonrelativistic case, when $\mathbf{v}^2 \ll c^2$ it follows from (13.19) that

$$\dot{v}_{\|} = |\boldsymbol{\nabla} V| \pm \sqrt{|\boldsymbol{\nabla} V|^2 - \langle \dot{\mathbf{v}}_{\perp}^2 \rangle}, \quad \langle \dot{\mathbf{v}}_{\|} \rangle = \boldsymbol{\nabla} V \tag{13.20}$$

where $\dot{\mathbf{v}}_{\|}$ and $\dot{\mathbf{v}}_{\perp}$ are components of the acceleration $\dot{\mathbf{v}}$, which are parallel and perpendicular to $\boldsymbol{\nabla} V$. The symbol $\langle . \rangle$ means averaging. In the relativistical case $\dot{\mathbf{v}}$ depends on $v^\alpha v^\beta \partial_\alpha \partial_\beta V$. This example shows that the skeleton conception of dynamics has more capacities for description of dynamics.

Logical reloading in the particle dynamics and in the Euclidean geometry admits one to construct such a conception of particle dynamics, which differs essentially from the conventional conception of particle dynamics created by Lagrange and Euler.

14 Inconceivable efficiency of the logical reloading

Two considered examples of the logical reloading show inconceivable efficiency of this very simple procedure. A simple replacement of a deterministic particle by the statistical ensemble of deterministic particles admits one to construct unified dynamic formalism of deterministic and stochastic particles. It appears that quantum particles are stochastic particles. Quantum mechanics is founded as a dynamics of statistical ensembles of stochastic particles.

Considering final result one can state, that transition to unified formalism of the arbitrary particle description is realized as a result of logical reloading in conception of the classical dynamics of deterministic particles. This transition is followed by a change of mathematical formalism. Statistical ensemble is considered as a dynamic system, consisting of infinite number of independent identical particles. As a result the statistical ensemble is considered as a continuous medium. Such a consideration differs from the case, when the statistical ensemble is described by the distribution function in the phase space. In transition to description in terms of wave function one uses integration of three equations (4.5), describing motion of a particle in the given field of velocities. The velocities in (4.5) are given indirectly by equations (4.3) (4.4). Such a way of integration looks rather unexpectedly. The new mathematical formalism generates some difficulties for its perception. As a result the key points of the logical reloading appear to be connected with details of the mathematical formalism.

One should note, that the dynamics of stochastic ensembles is a more general conception, than the quantum mechanics, which is only a special case the stochastic particles dynamics. Indeed, the stochastic particle dynamics can be described in terms of the wave function. In the nonrelativistic case we obtain the equation (3.8). In the relativistic case one has (5.32). Both equations (3.8) and (5.32) describe the general case of the stochastic particle motion. These equations described in terms of the wave function are nonlinear, generally speaking . They become linear and coincide with quantum description (3.7) and (5.31) respectively only in the case, when the flow of "quantum fluid" in the statistical ensemble is nonrotational. In this case the quantum principle of linearity appears. Besides, in the conception of the stochastic particle motion there is no such a problem as the problem of uniting of the quantum principles with the relativity theory. It is sufficient to consider relativistic Lagrangian for the statistical ensemble, to obtain "this uniting". But the most important feature of the statistical description of stochastic particles is the fact, that this conception leads to structural approach to elementary particles. Quantum principles do not permit one to obtain structural approach to elementary particles, because the quantum mechanics realizes too rough approximation. They do not admit one to obtain the κ-field which is responsible for pair production. The discovery of the κ-field is the first step to the structural approach. Quantum principles do not admit one to obtain helical shape of the electron world line and to explain appearance of spin and magnetic moment.

Madelung [5] and Bohm [31] used hydrodynamics for description of quantum phenomena, but they started from the Schrödinger equation and quantum principles. They consider only nonrotational motion of the "quantum fluid". They could not go outside the quantum principles and could not obtain quantum mechanics as a special case of a more general conception.

The main reason of appearance of the statistical foundation of quantum mechanics was consideration of the statistical ensemble as a dynamic system. In the nonrelativistic description of stochastic particles one uses usually statistical ensembles, but it is described by means of distribution function $F(t, \mathbf{x}, \mathbf{p})$, which describes a distribution of the statistical ensemble particles in the phase space of coordinate and momenta. At such a description the statistical ensemble was not a dynamic system. Such a description was not a relativistic description, because the concept of the phase space is not a relativistic concept. Nonrelativistic quantum mechanics is a relativistic conception, because only the mean motion of stochastic particles is nonrelativistic. The stochastic component of the particle velocity may by relativistic, and the statistical ensemble should be described relativistically. From practical viewpoint the description of statistical ensemble as a dynamic system means that *the concept of probability is not used in description of the statistical ensemble.*

Relativistic approach to description of the statistical ensemble was realized in papers [32, 33, 34] without the logical reloading. The statistical ensemble of stochastic particles was described as a dynamic system (continuous medium). As a result the statistical foundation of quantum mechanics as a statistical description of the stochastic particles motion has been derived.

As it concerns the logical reloading, it totalizes only the situation: transition to unified formalism of dynamics of deterministic and stochastic particles has been produced on the fundamental level, and it is produced in the framework of the good old classical dynamics. This transition was carried out by means of a simple logical reloading. *No additional hypotheses were used at such a transition.* Nevertheless, the result of such simple procedure as logical reloading is impressive. It is used to think usually that quantum principles are prime physical principles of the nature, and it is difficult to accept the viewpoint, when quantum principles are secondary principles generated by statistical description of the stochastic particles motion.

After transition to statistical foundation of quantum mechanics the problem arose. Why do free elementary particles move stochastically? The discrete space-time geometry in microcosm was the answer to this question. Idea of discrete space-time geometry is an old idea. Unfortunately, we did not know, how to describe a discrete geometry. One believes that the discrete geometry is a geometry on a lattice. Idea of the physical geometry, known as the distance geometry [35, 25] is very old. However, the mathematical aspect of the distance geometry was not developed properly. One did not know how one should use distance for construction of geometrical objects and geometrical relations. One tried to use methods of differential geometry, but it was not effective, because methods of differential geometry are adequate only in continuous geometries. The distance geometry is not sensitive to continuity. It may be used for both continuous and discrete geometries. As a result the methods of differential geometry are not effective in application to the distance geometry.

Any geometry is constructed as a generalization of the proper Euclidean geometry. The Euclidean geometry has two independent aspects: (1) geometry as a science on the shape geometrical objects and their mutual disposition (physical geometry) and (2) geometry as a logical construction (mathematical geometry). These two aspects are independent. Describing space-time, one uses the physical geometry, which is described completely by the unique quantity – distance. In this case it is of no importance, whether or not the geometry is a logical construction. The Euclidean geometry is a mathematical geometry and the physical geometry at once. Constructing Euclidean geometry as a logical construction (mathematical geometry), one obtains a geometry as a science on properties of geometrical objects (physical geometry). Working during two thousand years only with the Euclidean geometry, scientists did not differ between the two independent aspects of a geometry. But only Euclidean (logical) method of the geometry construction was known. It was used for construction of any geometry, because the method of the physical geometry construction was not known. Independence of two aspects of a geometry was not known also.

Situation changed essentially, when the direct method of the physical geometry construction (deformation principle [23]) has been obtained. The proper Euclidean geometry is presented in the form of a physical geometry, i.e. all relations and geometrical objects are described in terms of the Euclidean world function σ_{E} (or in terms of Euclidean distance). Thereafter the Euclidean world function σ_{E} is replaced by the world function σ of the physical geometry $\mathcal{G}$. As a result one obtains all relations of the geometry $\mathcal{G}$. It was of no importance, whether or not the geometry $\mathcal{G}$ may be considered as a logical construction.

Presentation of the Euclidean geometry in terms of one quantity – world function is a logical reloading in the formulation of the Euclidean geometry. The Euclidean geometry becomes to be a monistic conception, which can be transformed to any physical geometry by means of a simple replacement of the world function. Any physical geometry (except for Euclidean one) can be obtained by means of the deformation principle.

As a result the number of possible space-time geometries increase. The space-time geometry appeared to be multivariant and discrete in microcosm. Such a space-time geometry explains stochastic motion of free elementary particles. Appearance of non-Riemannian space-time geometries leads to extension of the general relativity on non-Riemannian space-time geometry. In the extended general relativity the induced antigravitation appears, which prohibits from formation of black holes [1, 2].

Consideration of discrete space-time geometry admits one to produce the physics geometrization and to construct the skeleton conception of elementary particles [29]. The skeleton conception (SC) realizes the structural approach to the elementary particle theory. It admits one to determine structure and arrangement of elementary particles. For instance, discovery of the κ-field is an appearance of the structural approach. This approach is based on physical principles and on a use of minimal number of fundamental concepts, whereas the standard model (SM) of elementary particles is based on empirical approach and on a use of experimental data. Interrelation of the skeleton conception with the standard model reminds interrelation of atomic physics with the periodical system of chemical elements. The two conceptions (SC and SM) do not contradict each other. It is only two different views on the theory of elementary particles. For instance, from viewpoint of the skeleton conception the quarks are simply elements of the hadron structure, but from the viewpoint of standard model the quarks are elementary particles.

References

[1] Yu. A. Rylov, General relativity extended to non-Riemannian space-time geometry. *e-print /0910.3582v7*

[2] Yu. A. Rylov, Induced antigravitation in the extended general relativity. *Gravitation and Cosmology*, Vol. **18**, No. 2, pp. 107–112, (2012). DOI: 10.1134/S0202289312020089

[3] Yu.A.Rylov, Spin and wave function as attributes of ideal fluid. *J. Math. Phys.* **40**, No.1, 256-278, (1999).

[4] Yu. A. Rylov, Uniform formalism for description of dynamic quantum and stochastic systems. *e-print /physics/0603237v6*

[5] E. Madelung, *Z.Phys.* **40,** (1926), 322.

[6] Yu.A.Rylov, Quantum mechanics as a dynamic construction. *Found. Phys.* **28**, No.2, 245-271, (1998).

[7] C.C. Lin, *Proc. International School of Physics "Enrico Fermi".* Course XXI, Liquid Helium , New York, Academic. 1963, pp. 93-146.

[8] A. Clebsch, *J. reine angew. Math.* **54** , 293, (1857).

[9] A. Clebsch, *J. reine angew. Math.* **56** , 1, (1859).

[10] Yu.A.Rylov, Classical description of pair production. *e-print /physics/0301020.*

[11] J. Glimm and A. Jaffe, *Phys. Rev.* **176** (1968) 1945.

[12] J. Glimm and A. Jaffe, *Ann. Math.* **91** (1970) 362.

[13] J. Glimm and A. Jaffe, *Acta Math.* **125** (1970) 203.

[14] J. Glimm and A. Jaffe, *J. Math. Phys.* **13** (1972) 1568.Liquid Helium , New York, Academic. 1963, pp. 93-146.

[15] Yu.A. Rylov, On connection between the energy-momentum vector and canonical momentum in relativistic mechanics. *Teoretischeskaya i Matematischeskaya Fizika.* **2**, 333-337 (1970.(in Russian). *Theor. and Math. Phys. (USA)*, **5**, 333, (1970) (trnslated from Russian)

[16] Yu.A. Rylov, On quantization of non-linear relativistic field without recourse to perturbation theory. *Int. J. Theor. Phys.* **6**, *181-204, (1972).*

[17] Yu.A. Rylov, Dirac equation in terms of hydrodynamic variables , *Advances Appl. Clifford Algebras.* **5**, No. 1, 1-40, (1995)

[18] Yu.A. Rylov, Statistical ensemble technique in application to description of the electron. *Advances Appl. Clifford Algebras* **7**(S), *216-228, (1997).*

[19] Yu. A. Rylov, Is the Dirac particle composite? *e-print /physics/0410045.*

[20] Yu. A. Rylov, Is the Dirac particle completely relativistic? *e-print /physics/0412032.*

[21] Yu.S.Vladimirov, *Geometrodynamics*, chpt. 8, Moscow, Binom, 2005 (in Russian)

[22] Yu.A. Rylov, Geometry without topology as a new conception of geometry. *Int. Jour. Mat. & Mat. Sci.* **30**, iss. 12, 733-760, (2002), see also *e-print /math.MG/0103002.*

[23] Yu. A.Rylov, Deformation principle and further geometrization of physics. *e-print* /0704.3003

[24] Yu.A. Rylov, Non-Euclidean method of the generalized geometry construction and its application to space-time geometry in *Pure and Applied Differential geometry* pp.238-246. eds. Franki Dillen and Ignace Van de Woestyne. Shaker Verlag, Aachen, 2007. See also *e-print Math.GM/0702552*

[25] L.M. Blumenthal, *Theory and Applications of Distance Geometry*, Oxford, Clarendon Press, 1953

[26] Yu. A. Rylov, Dynamic equations for tachyon gas, *Int. J. Theor. Phys.* **52**, 133(10), 3683- 3695, (2013),. doi:10.1007/s10773-013-1674-4

[27] Yu. A. Rylov, Different conceptions of Euclidean geometry and their application to the space-time geometry. *e-print /0709.2755v4*

[28] Yu. A.Rylov, Discrimination of particle masses in multivariant space-time geometry. *e-print /0712.1335.*

[29] Yu. A.Rylov, Discrete space-time geometry and skeleton conception of particle dynamics. *Int. J. Theor. Phys.* **51**, *Issue 6, 1847-1865, (2012).* See also *e-print* /1110.3399v1

[30] Yu. A.Rylov, Physics geometrization in microcosm: discrete space-time and relativity theory (Review). *Hypercomplex numbers in physics and geometry* **8**, i*ss. 2 (16,) pp.88-117 (2011).* In Russian. See also *e-print /1006.1254v2*

[31] D.Bohm, *Phys. Rev.* **85**, (1952), 166, 180

[32] Yu.A.Rylov, Quantum Mechanics as a theory of relativistic Brownian motion". *Ann. Phys. (Leipzig).* **27**, *1-11, (1971).*

[33] Yu.A.Rylov, Quantum mechanics as relativistic statistics.I: The two-particle case. *Int. J. Theor. Phys.* **8**, *65-83.*

[34] Yu.A.Rylov, Quantum mechanics as relativistic statistics.II: The case of two interacting particles. *Int. J. Theor. Phys.* **8**, *123-139.*

[35] K. Menger, Untersuchen über allgemeine Metrik, *Mathematische Annalen,* **100**, 75-113, (1928).