Incompleteness of the particle dynamics in microcosm and the skeleton conception of elementary particles as overcoming of this incompleteness

Yuri A.Rylov

Institute for Problems in Mechanics, Russian Academy of Sciences,
101-1, Vernadskii Ave., Moscow, 119526, Russia.
e-mail: rylov@ipmnet.ru
Web site: $http://rsfq1.physics.sunysb.edu/\tilde{}rylov/yrylov.htm$
or mirror Web site:
$http://gasdyn-ipm.ipmnet.ru/\tilde{}rylov/yrylov.htm$

Abstract

The skeleton conception of elementary particles is considered in the paper. Conventional particle dynamics is formulated in an unaccomplished form, which is adequate only in the continuous space-time geometry. The conventional differential equations of the particle motion cannot be written in the discrete space-time geometry. In the discrete space-time geometry the particle world line is replaced by the world chain. The world chain links has a finite length (not infinitesimal). The world chain appears to be stochastic. Statistical description of stochastic world chains leads to the Schrödinger equation, if the elementary length of the geometry is chosen in a proper way. The quantum principles are founded by existence of the discrete (and multivariant) space-time geometry and lose the role of prime physical principles. In the skeleton conception the particle is described by its skeleton (several rigidly connected space-time points). Skeleton conception of the elementary particles realizes a proper description of the particle state, which appears to be adequate in a discrete space-time geometry. The particle dynamics takes the form of a monistic conception, which is described completely in terms of the world function of the space-time geometry. The skeleton conception accomplishes the transition from nonrelativistic physics to the relativistic one and realizes the total geometrization of particle dynamics.

1 Introduction

A conception of the elementary particles is the first stage of the elementary particle theory. The conception considers concepts of the theory, logical connection between the concepts, their compatibility between themselves and with the relativity principles. Some concrete statements of the conception (for instance, a choice of the space-time geometry) are not formulated. As a result the conception of elementary particles cannot be tested experimentally, because it cannot make any concrete predictions. At the next step of the theory development, when a concrete space-time geometry is chosen, the theory can make predictions, which can be tested experimentally. Consideration of a conception of elementary particles is interesting in the relation, that it may show, *which concepts cannot be used in the elementary particle theory.*

Now, hundred years after construction of the special relativity the statement, that the special relativity theory is not yet accomplished, looks very strange and unexpected. However, transition from nonrelativistic physics to the relativistic one concerns mainly dynamic equations, describing particle motion. Concept of the particle state remains to be nonrelativistic, and this circumstance leads to many undesirable consequences. In particular, the particle state, which is defined as a particle momentum, given at some space-time point, is a concept of nonrelativistic physics. Concept of the phase space of the particle positions and momenta is also a nonrelativistic concept. The relativistic particle state is given in the whole space-time. It is given by the world line of the particle. Such a concept of the particle state has been introduced in [1]. As a result, ignoring the phase space and using relativistic concept of the particle state, one succeeded to obtain the quantum description as a statistical description of stochastic classical particles. A use of the phase space does not admit one to derive such a description [2, 3].

In this paper we ignore the phase space and use sequentially the relativistic concept of the particle state. Such an approach admits one to construct skeleton conception of elementary particles, which realizes a consistent relativistic dynamics of classical particles in the discrete space-time geometry.

In non-relativistic physics the state of a physical system is defined as a set of quantities which are given at a certain moment of time. Equations of motion determine these quantities at any subsequent moment of time. They describe the time evolution of the system state. The state and the equations of motion describing the time evolution of the state are two essential elements of any non-relativistic physical theory.

As it follows from the definition, a state of a system is given at a certain time moment. But in relativistic theory a simultaneity is relative. Which events are synchronous and which are not, it depends on the choice of a frame of reference. If, for example, one knows a state of a physical system in a frame of reference K, one could describe the state in a frame of reference K' moving relative to K only in the case, when the equations of motion are known and they can be solved. Thus, in the relativistic theory the state and the equations of motion are connected closely. As far as there is no absolute simultaneity in the relativistic theory, it seems more consistent to define the state of a system not at a given moment but over all space-time. In this case the concept of state will include the law of evolution of the physical system. The equations of motion are treated now as constraints imposed on the admissible states of the considered system.

Not all possible states are realized. Only those states are realized, which satisfy certain equations. We shall call them the constraint equations. In reality they are the same equations of motion but now they do not describe the time evolution of the state but they are restrictions which choose the physically allowable states from all virtual ones.

In short, in the non-relativistic theory the unique division of the physical phenomena description into states and equations of motion corresponds to the unique division of space-time into space and time. In the relativistic theory, where the division of space-time into space and time is conventional and not unique, the division of the physical phenomena description into states and equations of motion is not unique either. The physical system state defined over all space-time corresponds much better to the indivisible space-time.

The manner of division of the physical system description into states and equations of motion is unimportant for the dynamics of deterministic particles, but it is important for dynamics of indeterministic particles, when one uses a statistical description. Any statistics is a calculus of states. It is important for statistics what is understood under "state". In general, a statistics that corresponds to a different division of the description of a physical system into states and equations of motion leads to different results. The concept of the state density is the main concept of a statistical description.

In the nonrelativistic physics the state density ρ is defined by the relation

$$dN = \rho dV \tag{1.1}$$

where the state density ρ is the proportionality coefficient between the infinitesimal 3-volume dV and the number dN of particles in this 3-volume. In the nonrelativistic case the state density ρ is either 3-scalar, or a time component of some timelike 4-vector. Besides, the nonrelativistic state density ρ is nonnegative. At a proper normalization the state density ρ may be interpreted as a probability density of the particle detection at some space point.

In the relativistic physics the state density j^k at some space-time point x is defined as a proportionality coefficient j^k between the infinitesimal 4-vector area dS_k and the flux dF of oriented world lines, crossing this area

$$dF = j^k dS_k \tag{1.2}$$

In the relativistic case the state density is a 4-vector j^k, $k = 0, 1, 2, 3$. Its time component j^0 is positive for a particle and it is negative for an antiparticle. The time component j^0 cannot be interpreted as a probability density, because j^0 may have any sign. If $j^0(x) = 0$, it does not mean, that the number of particle at the point x is equal to zero. It means only, that the number of particles and the number of antiparticles at the point x are equal.

In general, a use of nonrelativistic terms at description of relativistic particles leads to misunderstandings. In the nonrelativistic physics a pointlike particle $\mathcal{P}$ (a point in 3-space) is considered as a real physical object, whereas its world line $\mathcal{L}$ is considered as a property of the pointlike particle $\mathcal{P}$ (its history). In the relativistic physics the world line $\mathcal{L}$ is a physical object, whereas the pointlike particle $\mathcal{P}$ is a property of the physical object $\mathcal{L}$ (intersection of $\mathcal{L}$ with the plane $x^0 = \text{const}$).

It is useful to introduce a special term for the physical object $\mathcal{L}$. We use the term "emlon". It is a reading of abbreviation "ML" of Russian term world line (WL). The point $\mathcal{P}$ of intersection of the emlon with the surface $x^0 = \text{const}$ is called semlon (or esemlon). It is a reading of abbreviation SML, which means in Russian "section of world line". Semlon is a collective concept with respect to concepts of particle and antiparticle. Particle and antiparticle are two different states of semlon. In the nonrelativistic physics particle and antiparticle are considered usually as two different physical objects, and this circumstance is important in the quantum field theory.

The second quantization of a scalar field is produced usually in terms of particles and antiparticles, which are considered as independent physical objects. It leads to nonstationary vacuum state, virtual particles, a use of a perturbation theory and other exotic results. The second quantization in terms of emlons (in terms of world lines, considered as physical objects) leads to a stationary vacuum and to a possibility of a quantization without recourse to perturbation theory [5].

In general, in the framework of the relativistic physics one uses sometimes nonrelativistic language, and this prevents from a consistent use of relativistic physics. For instance, one says: "World line of a particle and world line of its antiparticle disappear at a collision." From consequent relativistic viewpoint the same statement should be presented as follows: "If emlon changes its directivity in the time direction, then one branch of the emlon describes a particle, whereas another branch describes an antiparticle." It is different mathematical technique, which is placed behind the two expressions, describing the same situation.

In the nonrelativistic physics the state of a pointlike particle is given as a point in the phase space. The particle position and the particle momentum, are given at some time moment, and they describe the particle state. Besides, one ascribes to the particle its mass and its charge. Such a description of the pointlike particle state is the same in the nonrelativistic theory and in the relativistic one.

A use of the phase space supposes, that the particle motion is described by a smooth continuous world line $x^k = x^k(\tau)$, $k = 0, 1, 2, 3$ and τ is a parameter along the world line. It supposes, that the momentum p_k can be defined as limit

$$p_k(\tau) = \frac{m g_{kl} u^l(\tau)}{\sqrt{g_{js} u^j(\tau) u^s(\tau)}}, \qquad u^l(\tau) = \lim_{d\tau \to 0} \frac{x^l(\tau + d\tau) - x^l(\tau)}{d\tau} = \frac{dx^l(\tau)}{d\tau} \tag{1.3}$$

If we consider a particle motion in the microcosm, we cannot be sure, that the world line is smooth and continuous. Discrete space-time geometry may violate smoothness of the world line. Furthermore, it is known from experiments with microparticles (electrons and elementary particles) that their motion is stochastic, i.e. it cannot be described by a smooth continuous world line. It is of no importance, whether such a motion is explained by a discrete space-time geometry or by the quantum nature of microparticle. In these cases the limit (1.3) does not exist, and one cannot introduce the phase space, founded on a use of the limit (1.3).

In the case of indeterministic particles we are forced to refuse from a use of the limit (1.3) and define the pointlike particle state by two points P_0, P_1 in the space-time. The vector $\mathbf{P}_0\mathbf{P}_1$ is an ordered set $\{P_0, P_1\}$ of two points P_0, P_1. The vector $\mathbf{P}_0\mathbf{P}_1$ describes a geometrical momentum of the particle, and its length $\mu = |\mathbf{P}_0\mathbf{P}_1|$ is its geometrical mass. The usual 4-momentum $\mathbf{p}$ and the usual mass m of the particle are connected with geometrical quantities by the relations

$$\mathbf{p} = bc\mathbf{P}_0\mathbf{P}_1, \qquad m = b\mu = b\,|\mathbf{P}_0\mathbf{P}_1|\,, \qquad g^{kl}p_kp_l = m^2c^2 \tag{1.4}$$

where b is some universal constant, and c is the speed of the light.

Such a generalization of concept of a world line follows from the fact, that in the proper Euclidean geometry a smooth line is defined as a limit of the broken straight line, when length of its straight links tends to zero. If the limit (1.3) does not exist, we are forced to use the broken line instead of smooth line. In other words, we are forced to use the world chain instead of a world line.

Experiments show, that the elementary particle motion in the microcosm is stochastic, and the elementary particles are indeterministic particles. In this case the limit (1.3) cannot be used, and an indeterministic pointlike particle is described by the world chain $\mathcal{C}$ (instead of the world line)

$$\mathcal{C} = \bigcup_s \mathbf{P}_s\mathbf{P}_{s+1} \qquad |\mathbf{P}_k\mathbf{P}_{k+1}| = |\mathbf{P}_{k+1}\mathbf{P}_{k+2}| \quad k = ... - 1, 0, 1, ... \tag{1.5}$$

The world chain is described as a set of points $\{P_s\}$. It is of no importance, whether there are another points between the points P_s and P_{s+1}, which belong to the chain. In other words, it is unessential, whether the world chain is used in a continuous geometry or in a discrete one. Such a definition of the pointlike particle state can be used in the case of possible discreteness of the space-time geometry. Besides, this definition does not contain a reference to a coordinate system. This definition does not need an existence of the limit (1.3). World chain may contain links of a finite length. In such a situation one cannot introduce concept of the phase space in that form, which is used in the nonrelativistic physics, where the space-time geometry is considered to be continuous.

If the link length is small enough, from macroscopic viewpoint, it can be considered as infinitesimal, the world chain can be approximated by a continuous world line, and the mass can be ascribed to the particle world line as an external parameter. In this case one can return to the case (1.3), and the concept of the phase space can be introduced.

If the particle is free, the adjacent links of the world chain are equivalent (equal), and the point P_{s+2} is defined via the points P_s, P_{s+1} by means of equations

$$2\left|\mathbf{P}_s\mathbf{P}_{s+1}\right| = \left|\mathbf{P}_s\mathbf{P}_{s+2}\right|, \quad \left|\mathbf{P}_s\mathbf{P}_{s+1}\right| = \left|\mathbf{P}_{s+1}\mathbf{P}_{s+2}\right|, \qquad s = 0, \pm 1, \pm 2, ... \tag{1.6}$$

In the space-time of Minkowski the equations (1.6) has a unique solution for the point P_{s+2}, provided all links are timelike.

In the discrete (physical) space-time geometry, where geometry is defined completely by the world function $\sigma(P_0, P_1) = \frac{1}{2}\left|\mathbf{P}_0\mathbf{P}_1\right|^2$, the equations (1.6) has many solutions, in general, even for timelike links. In this case the phase space cannot be introduced, because the limit (1.3) does not exist. In other words, one cannot use nonrelativistic concept of the particle state in the microcosm.

The rule (1.6) of the world chain construction for a free particle coincides with the rule of straight line construction by means of a compass in the proper Euclidean geometry. In the discrete space-time geometry the rule (1.6) is formulated as follows. The adjacent links are equivalent $\mathbf{P}_s\mathbf{P}_{s+1}\text{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}$.

In the proper Euclidean geometry $\mathcal{G}_{\rm E}$ the equivalence of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined as follows. Vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent ($\mathbf{P}_0\mathbf{P}_1\text{eqv}\,\mathbf{Q}_0\mathbf{Q}_1$), if vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are in parallel ($\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1$) and their lengths $\left|\mathbf{P}_0\mathbf{P}_1\right|$ and $\left|\mathbf{Q}_0\mathbf{Q}_1\right|$ are equal. Mathematically the two conditions are written in the form

$$(\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1): \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \left|\mathbf{P}_0\mathbf{P}_1\right| \cdot \left|\mathbf{Q}_0\mathbf{Q}_1\right| \tag{1.7}$$

$$\left|\mathbf{P}_0\mathbf{P}_1\right| = \left|\mathbf{Q}_0\mathbf{Q}_1\right|, \qquad \left|\mathbf{P}_0\mathbf{P}_1\right| = \sqrt{2\sigma(P_0, P_1)} \tag{1.8}$$

where $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ is the scalar product of two vectors. It is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) \tag{1.9}$$

Here σ is the world function of the proper Euclidean geometry $\mathcal{G}_{\rm E}$. The length $|\mathbf{PQ}|$ of vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{PQ}| = \rho(P, Q) = \sqrt{2\sigma(P, Q)} \tag{1.10}$$

The definition (1.9) contains neither coordinate system, nor dimension of the space, and it may be used in any physical geometry, i.e. in the geometry described by the world function completely.

Using relations (1.7) - (1.10), one can write the equivalence condition in the form

$$\begin{aligned} \mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1 \quad &: \qquad \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) = 2\sigma(P_0, P_1) \\ \wedge\sigma(P_0, P_1) \quad &= \quad \sigma(Q_0, Q_1) \end{aligned} \tag{1.11}$$

If $P_0 = P_s$, $Q_0 = P_1 = P_{s+1}$ and $Q_1 = P_{s+2}$, the relations (1.11) take the form

$$\mathbf{P}_s\mathbf{P}_{s+1}\text{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}: \quad \sigma(P_s, P_{s+2}) = 4\sigma(P_s, P_{s+1}) \wedge \sigma(P_s, P_{s+1}) = \sigma(P_{s+1}, P_{s+2}) \tag{1.12}$$

which coincides with (1.6)

The equivalence relation (1.11) is used in any physical geometry.

2 Statistical description of indeterministic particles

In the beginning of the twentieth century it was natural to think, that the quantum particles are simply indeterministic (stochastic) particles, something like Brownian particles. There were attempts to obtain quantum mechanics as a statistical description of stochastically moving particles [2, 3]. However, these attempts failed because a *probabilistic conception of the statistical description* was used.

Statistical description is used in physics for description of indeterministic particles (or systems), when there are no dynamic equations, or initial conditions are indefinite. One considers statistical ensemble of indeterministic particles, i.e. many independent similar particles. It appears, that there are dynamic equations for the statistical ensemble $\mathcal{E}$ of indeterministic particles, although there are no dynamic equations for a single indeterministic particle, which is a constituent of this statistical ensemble $\mathcal{E}$. Consideration of the statistical ensemble as a dynamic system is a *dynamic conception of the statistical description* (DCSD). It is a primordial conception of statistical description. A use of DCSD is founded on independence of constituents of the statistical ensemble. Random components of motion are compensated due to their independence, whereas regular components of motion are accumulated.

In the nonrelativistic physics *the probabilistic conception of the statistical description* (PCSD) is used. PCSD is used successfully, for instance, for description of Brownian motion. In PCSD one traces the motion of the point in the phase space. The point represents the state of indeterministic particle, and a motion the point in the phase space is described by the probability transition. Attempts of obtaining the quantum mechanics as a result of description in the framework PCSD failed, because PCSD is a nonrelativistic description, whereas the nonrelativistic quantum mechanics is a relativistic construction, and the quantum mechanics should be obtained as a statistical description in terms of DCSD.

But why is the nonrelativistic quantum mechanics a relativistic construction? Because the stochastic component of the quantum particle motion may be relativistic, and one has to use the dynamic conception of statistical description (DCSD), which does not use the nonrelativistic concept of the phase space.

Indeed, in terms of DCSD one succeeded to obtain the quantum mechanics as a statistical description of stochastically moving particles [1, 4, 6, 7]. The action for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm st}]$ of free indeterministic particles $\mathcal{S}_{\rm st}$ is written in the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm st}]}[\mathbf{x},\mathbf{u}] = \int\int_{V_\xi} \left\{ \frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u} \right\} dt d\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{2.1}$$

Independent variables $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$ label constituents $\mathcal{S}_{\rm st}$ of the statistical ensemble. The dependent variable $\mathbf{x} = \mathbf{x}(t,\boldsymbol{\xi})$ describes the regular component of the particle motion. The variable $\mathbf{u} = \mathbf{u}(t,\mathbf{x})$ describes the mean value of the stochastic velocity component, $\hbar$ is the quantum constant. The second term in (2.1) describes the kinetic energy of the stochastic velocity component. The third term describes interaction between the stochastic component $\mathbf{u}(t,\mathbf{x})$ and the regular component $\dot{\mathbf{x}}(t,\boldsymbol{\xi})$. The operator

$$\nabla=\left\{\frac{\partial}{\partial x^1},\frac{\partial}{\partial x^2},\frac{\partial}{\partial x^3}\right\} \tag{2.2}$$

is defined in the space of coordinates $\mathbf{x}$. Dynamic equations for the dynamic system $\mathcal{E}[\mathcal{S}_{\text{st}}]$ are obtained as a result of variation of the action (2.1) with respect to dynamic variables $\mathbf{x}$ and $\mathbf{u}$.

The action for a single indeterministic particle $\mathcal{S}_{\text{st}}$ has the form

$$\mathcal{A}_{\mathcal{S}_{\text{st}}}[\mathbf{x},\mathbf{u}]=\int\int_{V_\xi}\left\{\frac{m}{2}\dot{\mathbf{x}}^2+\frac{m}{2}\mathbf{u}^2-\frac{\hbar}{2}\nabla\mathbf{u}\right\}dt,\qquad \dot{\mathbf{x}}\equiv\frac{d\mathbf{x}}{dt} \tag{2.3}$$

This action is not correctly defined, because operator ∇ is defined on 3D-space of coordinates $\mathbf{x}=\{x^1,x^2,x^3\}$, whereas in the action functional (2.3) the variable $\mathbf{x}$ is used only on one-dimensional set. It means that there are no dynamic equations for the particle $\mathcal{S}_{\text{st}}$, and the particle $\mathcal{S}_{\text{st}}$ is a stochastic (not dynamic) system. However, the action functional (2.1) is well defined, and dynamic equations exist for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\text{st}}]$, although dynamic equations do not exist for constituents of this statistical ensemble.

Variation of the action (2.1) leads to dynamic equations

$$\delta\mathbf{u}:\qquad m\rho\mathbf{u}+\frac{\hbar}{2}\nabla\rho=0,\qquad \mathbf{u}=-\frac{\hbar}{2m}\nabla\ln\rho \tag{2.4}$$

$$\delta\mathbf{x}:\qquad m\frac{d^2\mathbf{x}}{dt^2}=\nabla\left(\frac{m}{2}\mathbf{u}^2-\frac{\hbar}{2}\nabla\mathbf{u}\right) \tag{2.5}$$

where

$$\rho=\frac{\partial(\xi_1,\xi_2,\xi_3)}{\partial(x^1,x^2,x^3)}=\left(\frac{\partial(x^1,x^2,x^3)}{\partial(\xi_1,\xi_2,\xi_3)}\right)^{-1} \tag{2.6}$$

After proper change of variables the dynamic equations are reduced to the equation [7]

$$i\hbar\partial_0\psi+\frac{\hbar^2}{2m}\nabla^2\psi+\frac{\hbar^2}{8m}\nabla^2 s_\alpha\cdot(s_\alpha-2\sigma_\alpha)\psi-\frac{\hbar^2}{4m}\frac{\nabla\rho}{\rho}\nabla s_\alpha\sigma_\alpha\psi=0 \tag{2.7}$$

where ψ is the two component complex wave function

$$\rho=\psi^*\psi,\qquad s_\alpha=\frac{\psi^*\sigma_\alpha\psi}{\rho},\qquad \alpha=1,2,3 \tag{2.8}$$

σ_α are 2×2 Pauli matrices

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\qquad \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\qquad \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \tag{2.9}$$

If components ψ_1 and ψ_2 are linear dependent $\psi = \begin{pmatrix} a\psi_1 \\ b\psi_1 \end{pmatrix}$, $a, b = \text{const}$, then $\mathbf{s} = \text{const}$. Two last terms of the equation (2.7) vanish, and the equation turns to the Schrödinger equation

$$i\hbar\partial_0\psi + \frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi = 0 \tag{2.10}$$

Thus, the Schrödinger equation and interpretation of the quantum mechanics appear from the dynamical system $\mathcal{E}[\mathcal{S}_{\text{st}}]$, described by the action functional (2.1). This fact seems rather unexpected, because in quantum mechanics the wave function is considered as a specific quantum object, which has no analog in classical physics. In reality, the wave function is simply a way of description of ideal continuous medium [8]. You may describe an ideal fluid in terms of hydrodynamic variables: density ρ and velocity $\mathbf{v}$. You may describe an ideal fluid in terms of the wave function. The statistical ensemble $\mathcal{E}[\mathcal{S}_{\text{st}}]$ is a dynamic system of the type of continuous medium. The two representations of dynamic equations for the dynamic system $\mathcal{E}[\mathcal{S}_{\text{st}}]$ can be transformed one into another.

It is well known, that the Schrödinger equation can be written in the hydrodynamic form of Madelung-Bohm [9, 10]. The wave function ψ is presented in the form

$$\psi = \sqrt{\rho}\exp(i\varphi/\hbar) \tag{2.11}$$

Substituting (2.11) in the Schrödinger equation (2.10), one obtains two real equations for dynamical variables ρ and φ. Taking gradient from the equation for φ and introducing designation

$$\mathbf{v} = -\frac{\hbar}{m}\boldsymbol{\nabla}\varphi, \qquad \text{curl}\,\mathbf{v} = 0 \tag{2.12}$$

one obtains four equations of the hydrodynamical type

$$\frac{\partial\rho}{\partial t} + \boldsymbol{\nabla}(\rho\mathbf{v}) = 0, \qquad \frac{d\mathbf{v}}{dt} \equiv \frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\boldsymbol{\nabla})\,\mathbf{v} = -\frac{1}{m}\boldsymbol{\nabla}U_{\text{B}} \tag{2.13}$$

where U_{B} is the Bohm potential, defined by the relation

$$U_{\text{B}} = U\left(\rho, \boldsymbol{\nabla}\rho, \boldsymbol{\nabla}^2\rho\right) = \frac{\hbar^2}{8m\rho}\left(\frac{(\boldsymbol{\nabla}\rho)^2}{\rho} - 2\boldsymbol{\nabla}^2\rho\right) = -\frac{\hbar^2}{2m\sqrt{\rho}}\boldsymbol{\nabla}^2\sqrt{\rho} \tag{2.14}$$

Hydrodynamic equations (2.13) can be easily obtained from equations (2.4), (2.5). To obtain representation of equations (2.13), (2.14) in terms of wave function, one needs to integrate these equations, because they have been obtained by means of differentiation of the Schrödinger equation. This integration can be easily produced, if the condition (2.12) takes place and the fluid flow is non-rotational.

In the general case of vortical flow the integration is more complicated. Nevertheless this integration has been produced [8], and one obtains a more complicated equation (2.7), where two last terms describe vorticity of the flow. The Schrödinger equation (2.10) is a special case of the more general equation (2.7).

Note that the equation (2.7) is not linear, although it is invariant with respect to transformation

$$\psi \rightarrow \tilde{\psi} = A\psi, \qquad A = \text{const} \tag{2.15}$$

which admits one to normalize the wave function to any nonnegative quantity. This property describes independence of the statistical ensemble on the number of its constituents

Description of the pair production is obtained in the relativistic version of the action functional (2.1). This action has the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\text{st}}]}[x,\kappa] = -\int \int_{V_{\boldsymbol{\xi}}} mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k}\rho_0(\boldsymbol{\xi})\,d\tau d\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{d\tau} \tag{2.16}$$

$$K = \sqrt{1 + \lambda^2\left(g_{kl}\kappa^k\kappa^l + \partial_k\kappa^k\right)}, \qquad \lambda = \frac{\hbar}{mc} \tag{2.17}$$

where $x = \left\{x^k\right\} = \left\{x^k(\tau, \boldsymbol{\xi})\right\}$, $k = 0, 1, 2, 3$ are dependent variables. The quantity g_{kl} =diag$\{c^2, -1, -1, -1\}$ is the metric tensor. The independent variables $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$ label the particles of the statistical ensemble. The dependent variables $\kappa^k = \kappa^k(x)$, $k = 0, 1, 2, 3$ form some force field, connected with the stochastic component of the particle 4-velocity, and λ is the Compton wave length of the particle. Connection of the field κ^k with the mean value $u^k(t, \mathbf{x}) = u^k(x)$ of stochastic component of 4-velocity has the form

$$\kappa^k = \frac{m}{\hbar}u^k, \qquad k = 0, 1, 2, 3 \tag{2.18}$$

In the nonrelativistic approximation one may neglect the temporal component $\kappa^0 = \frac{m}{\hbar}u^0$ with respect to the spatial one $\boldsymbol{\kappa} = \frac{m}{\hbar}\mathbf{u}$. Setting $\tau = t = x^0$ in (2.16), (2.17), we obtain the action (2.1) instead of (2.16).

To describe the pair production, the world line is to have a possibility of turn in the time direction. At the turning point the world line has to be spacelike and the radical in (2.16) must be imaginary. It is possible, only if the quantity (2.17) is imaginary also. It means, the effective mass mK is to be imaginary. The quantity K may be imaginary, if the field κ^k have proper values. It means that the stochastic component of the particle motion is responsible for the pair production (turn of the world line in the time direction).

Representation of quantum mechanics as a statistical description of classical indeterministic particles admits one to interpret all quantum relations in terms of statistical description. This interpretation distinguishes in some clauses from the conventional interpretation of quantum mechanics.

In any statistical description there are two different kinds of measurement, which have different properties. Massive measurement (M-measurement) is produced over all constituents of the statistical ensemble. A result of M-measurement of the quantity R is a distribution of the quantity R, which can be predicted as a result of solution of dynamic equations for the statistical ensemble.

Single measurement (S-measurement) is produced over one of constituents of the statistical ensemble. A result of S-measurement of the quantity R is some random value of the quantity R, which cannot be predicted by the theory. In the Copenhagen interpretation of the quantum mechanics the wave function is supposed to describe a single particle (but not a statistical ensemble of particles). As a result there is only one type of measurement, which is considered sometimes as a M-measurement and sometimes as a S-measurement. As far as M-measurement and S-measurement have different properties, such an identification is a source of numerous contradictions and paradoxes [11].

Representation of quantum mechanics as a statistical description of the indeterministic particles motion has two important consequences: (1) elimination of quantum principles as laws of nature, (2) problem of primordial stochastic motion of free particles.

3 Multivariant space-time geometry as a corollary of existence of indeterministic particles.

Reduction of number of physical principles means an increase of the quality of the physical theory. Explanation of quantum effects by means of a stochasticity of free particle motion sets the question of the nature of this stochasticity. The *motion of a free particle is determined by properties of the space-time geometry*. The free particle motion is deterministic in the space-time of Minkowski. An indeterministic motion of free particles is possible only in multivariant space-time geometries. Such geometries were unknown in the twentieth century, and explanation of quantum effects by a stochasticity of particle motion seemed to be impossible.

The multivariant (physical) geometry is nonaxiomatizable, in general. It means, that statements of the multivariant geometry cannot be deduced from axiomatics. In the twentieth century only axiomatizable geometries were known. Mathematicians, who were responsible for investigation and creation of geometries, believed that any geometry is to be a logical construction. Hence, any geometry is to be axiomatizable.

In general, there were mathematicians [12, 13], who believed that the geometry may be a distance geometry, which is described by the distance function between any two points of the space. However, it was not known, how to construct geometrical objects in the distance geometry. The distance geometry appeared to be ineffective, and at description of the space-time the mathematicians ignored the distance geometry, as well as the metric geometry, which a special case of the distance geometry.

Situation was changed cardinally, when a way of the geometrical objects construction has been invented. It is the deformation principle [14]. One takes a geometrical object of the proper Euclidean geometry and describes it in terms of the Euclidean distance function ρ_{E} (or in terms of the Euclidean world function

$\sigma_{\rm E}=\frac{1}{2}\rho_{\rm E}^2$). Substituting Euclidean distance function $\rho_{\rm E}$ by the distance function ρ of the geometry in question $\mathcal{G}$, one obtains the geometrical object of the geometry $\mathcal{G}$. Although the deformation principle has been published in explicit form only in 2007, in fact it was used ab origine of the physical geometry construction [15].

As far as the formal logic is not used at the construction of geometric objects of the physical geometry, the obtained physical geometry is multivariant and non-axiomatizable, in general. It means, that solving equations (1.11) at given vector $\mathbf{P}_0\mathbf{P}_1$ and given point Q_0, one obtains, in general, many solutions for the vector $\mathbf{Q}_0\mathbf{Q}_1$. It is possible also such a situation, when equations (1.11) have no solution.

Multivariant space-time geometry made impossible a use of the limit (1.3) for a construction of the phase space, and nonrelativistic concept of the particle state becomes impossible for description of elementary particles. On the other hand, being a reason of the free particles motion stochasticity, the multivariant space-time geometry becomes to be interpreted as a reason of quantum effects [16]. Let us stress, that the obtaining of the Schrödinger equation as corollary of the multivariant space-time geometry appeared to be possible only at a use of the relativistic concept of the pointlike particle state (1.4), (1.5). Only in this case the free particle motion depends on the particle mass. Indeed, describing a free particle motion, the Schrödinger equation contains the particle mass, whereas the classic deterministic motion of a free particle is the same for particles of any mass. The length of links (1.4) of the world chain (1.5) is essential for the stochastic component of the particle motion.

4 Skeleton conception of elementary particles

After the paper [16] the role of the space-time geometry increased in the theory of elementary particles, because in fact the quantum principles were replaced by the multivariant space-time geometry. It became clear, that constructing a theory of elementary particles, one should use relativistic concept of the particle state.

In the case, when the particle is not pointlike, its state is described by its skeleton $\mathcal{P}_n=\{P_0,P_1,...,P_n\}$, which is a set of $(n+1)$ space-time points. $n>1$ is some integer number. These points are connected rigidly. In the case of a pointlike particle the skeleton consists of two points. The skeleton $\mathcal{P}_n$ is a natural generalization of the skeleton of the pointlike particle on the case of a composite particle. Motion of any particle is described by the world chain, consisting of connected skeletons [17]. $...\mathcal{P}_n^{(0)},\mathcal{P}_n^{(1)},...,\mathcal{P}_n^{(s)}...$

$$\mathcal{P}_n^{(s)}=\left\{P_0^{(s)},P_1^{(s)},..P_n^{(s)}\right\},\qquad s=...0,1,... \tag{4.1}$$

The adjacent skeletons $\mathcal{P}_n^{(s)},\mathcal{P}_n^{(s+1)}$ of the chain are connected by the relations $P_1^{(s)}=P_0^{(s+1)}$, $s=...0,1,...$ The vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}=\mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ is the leading vector, which determines the world chain direction.

Dynamics of a free elementary particle is determined by the relations

$$\mathcal{P}_n^{(s)} \text{eqv} \mathcal{P}_n^{(s+1)} : \qquad \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)} \text{eqv} \mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}, \qquad i,k = 0,1,..n; \qquad s = ...0,1,... \tag{4.2}$$

which describe equivalence of adjacent skeletons. Equivalence of vectors is defined by the relations (1.11).

Thus, dynamics of a free elementary particle is described by a system of algebraic equations (4.2). Specific of dynamics depends on the elementary particle structure (disposition of particles inside the skeleton) and on the space-time geometry. Lengths $\left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|$ of vectors $\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}$ are constant along the whole world chain. These $n(n+1)/2$ quantities may be considered as characteristics of the particle. In the case of pointlike particle the length $|\mathbf{P}_s\mathbf{P}_{s+1}|$ of the link $\mathbf{P}_s\mathbf{P}_{s+1}$ is the geometrical mass of the particle. In the case of a more complicated skeletons the meaning of parameters $\left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|$ should be investigated.

Remark. We are forced to reject from definition of particle state as some quantities given at some time moment, because some vectors of a skeleton are timelike, and one cannot find such a coordinate system, where all points of the skeleton have the same time coordinate. One cannot define the particle state as points of intersection of several world lines with the surface $x^0 = \text{const}$, because the space-time geometry may be discrete and continuous world lines do not exist.

The system of dynamic equations (4.2) consists of $n(n+1)$ algebraic equations for nD dynamic variables, where D is the dimension the space-time (the number of coordinates, which are necessary for labelling of all points of the space-time). If $n \leq D$, the number of dynamic variables is less, than the number of dynamic equations. In this case we have a discrimination mechanism, which forbids some skeletons. This mechanism admits one to explain discrete parameters of elementary particles. If $n > D+1$, the number of dynamic equations is more than the number of dynamic variables. In this case there may exist many solutions, and the particle motion becomes multivariant. Both cases may take place in the theory of elementary particles.

Dynamic equations (4.2) are written in the coordinateless form, and this fact is a worth of the dynamic equations (4.2), as far as it saves from a necessity to consider the coordinate transformations. Dynamic equations (4.2) are algebraic equations (not differential), and this fact is also a worth of the theory, because the algebraic equations may be used even in a discrete space-time geometry.

The first (nontrivial) attempt of a use of the relativistic concept of the particle state was made. One considered the structure of the Dirac particle (fermion) [18]. It appeared that the skeleton of the Dirac particle consists of n points ($n \geq 3$). Its world chain is a spacelike helix with a timelike axis. Spacelike world lines are impossible in the space-time geometry of Minkowski.

The Dirac particle is considered in the space-time geometry described by the world function σ_d

$$\sigma_\text{d} = \sigma_\text{M} + \lambda_0^2 \begin{cases} \text{sgn}(\sigma_\text{M}) & \text{if} \quad |\sigma_\text{M}| > \sigma_0 > 0 \\ f(\sigma_\text{M}) & \text{if} \quad |\sigma_\text{M}| < \sigma_0 \end{cases} \qquad \lambda_0^2 = \frac{\hbar}{2bc} \tag{4.3}$$

where σ_{M} is the world function of the space-time of Minkowski, b is some universal constant and σ_0 is some constant. The function f is a monotone nondecreasing function, having properties $f(-\sigma_0) = -1$, $f(\sigma_0) = 1$.

The space-time geometry, described by the world function (4.3) is uniform and isotropic. The part of the world function corresponding to $|\sigma_{\mathrm{M}}| > \sigma_0$ is responsible for quantum effects of a pointlike particle (Schrödinger equation [16]). The part of the world function (4.3), corresponding to $|\sigma_{\mathrm{M}}| < \sigma_0$ is responsible the structure of a particle with the skeleton consisting of more, than two points. If $|f(\sigma_{\mathrm{M}})| < |\sigma_{\mathrm{M}}/\sigma_0|$, the spacelike world chain may have a shape of a helix with a timelike axis.

The case, when

$$f(\sigma_{\mathrm{M}}) = \left(\frac{\sigma_{\mathrm{M}}}{\sigma_0}\right)^3 \qquad (4.4)$$

has been investigated. Such a choice of the world function does not pretend to description of the real space-time. It is only some model, which correctly describes quantum effects connected with pointlike particles and tries to investigate, whether spacelike world chain may have a shape of a helix with a timelike axis. According to semiclassical approximation of the Dirac equation [19, 20, 21] the world line of a *free classical* Dirac particle has the shape of a helix. Such a shape of the world line explains existence of a spin. It was interesting, whether the spin of the Dirac particle can be obtained in the skeleton conception of elementary particles.

Consideration in [18] confirmed the supposition on the helix world chain of the Dirac particle (fermion). The skeleton of a fermion is to contain more, than two points. Besides, some restrictions on disposition of the skeleton points were obtained. It means that in the skeleton conception there is a discrimination mechanism responsible for discrete values of parameters of the elementary particles. Such a discrimination mechanism is absent in the conventional approach, based on a use of quantum principles. The obtained results are preliminary, because the simple restriction (4.4) on the world function has been used. Nevertheless these results show, that the skeleton conception admits one to investigate the structure of elementary particles. The conventional approach, based on quantum principles, admits one only to ascribe to elementary particles such phenomenological properties as spin, color, flavour and other, without explanation how these properties relate to the elementary particle structure. The quantum approach admits one only to classify elementary particles by their phenomenological properties and to predict reaction between the elementary particles on the basis of this classification.

Such a situation reminds situation with investigation of chemical elements. Periodic system of chemical elements is a phenomenological construction. It is an attribute of chemistry. Arrangement of atoms of chemical elements is investigated by physics (quantum mechanics). The periodic system of chemical elements had been discovered earlier, than researchers began to investigate atomic structures. However, the periodic system did not help us to create quantum mechanics and to investigate the atomic structure. The periodic system and the quantum mechanics are attributes of different sciences. In the same track the skeleton conception of elementary particles and the conventional phenomenological approach based on quantum theory are essentially attributes of different sciences, investigating different sides of the elementary particles.

5 About conservation laws

Conservation laws of energy-momentum and angular momentum take place only in a uniform and isotropic space-time geometry. The real space-time geometry $\mathcal{G}_{\mathrm{r}}$, where all elementary particles move freely, is not uniform and isotropic, in general. To obtain the conservation laws, we may consider some fictitious space-time geometry $\mathcal{G}_{\mathrm{f}}$, which is uniform and isotropic. For instance, we may suppose, that the space-time geometry $\mathcal{G}_{\mathrm{f}}$ is the geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$. Let us describe the particle motion in the space-time geometry of Minkowski. The particle motion, which is free in the geometry $\mathcal{G}_{\mathrm{r}}$ ceases to be free in the fictitious geometry $\mathcal{G}_{\mathrm{M}}$. Some force fields appear. These force fields appear as a result of mismatch d between the world functions σ_{r} and σ_{M}

$$d\left(P,Q\right)=\sigma_{\mathrm{r}}\left(P,Q\right)-\sigma_{\mathrm{M}}\left(P,Q\right) \tag{5.1}$$

Rewriting dynamic equations (4.2) in terms of the world function σ_{M}, one obtains

$$\left(\mathbf{P}_{i}^{(s)}\mathbf{P}_{k}^{(s)}.\mathbf{P}_{i}^{(s+1)}\mathbf{P}_{k}^{(s+1)}\right)_{\mathrm{M}}=\left|\mathbf{P}_{i}^{(s)}\mathbf{P}_{k}^{(s)}\right|_{\mathrm{M}}^{2}-F\left(P_{i}^{(s)},P_{k}^{(s)},P_{i}^{(s+1)},P_{k}^{(s+1)}\right) \tag{5.2}$$

$$\left|\mathbf{P}_{i}^{(s)}\mathbf{P}_{k}^{(s)}\right|_{\mathrm{M}}^{2}=\left|\mathbf{P}_{i}^{(s+1)}\mathbf{P}_{k}^{(s+1)}\right|_{\mathrm{M}}^{2}-2d\left(P_{i}^{(s)},P_{k}^{(s)}\right)+2d\left(P_{i}^{(s+1)},P_{k}^{(s+1)}\right) \tag{5.3}$$

where

$$\begin{aligned}
& F\left(P_{i}^{(s)},P_{k}^{(s)},P_{i}^{(s+1)},P_{k}^{(s+1)}\right)\\
= & \ d\left(P_{i}^{(s)},P_{k}^{(s+1)}\right)+d\left(P_{k}^{(s)},P_{i}^{(s+1)}\right)-d\left(P_{i}^{(s)},P_{i}^{(s+1)}\right)-d\left(P_{k}^{(s)},P_{k}^{(s+1)}\right)\\
& -2d\left(P_{i}^{(s)},P_{k}^{(s)}\right)
\end{aligned} \tag{5.4}$$

In relations (5.2) - (5.4) indices $i,k=0,1,...n$ and s takes all integer values. The index "M" means that the scalar products are calculated in the geometry $\mathcal{G}_{\mathrm{M}}$ of Minkowski. The quantities $F\left(P_{i}^{(s)},P_{k}^{(s)},P_{i}^{(s+1)},P_{k}^{(s+1)}\right)$ describe some force fields, which appear in the geometry of Minkowski due to mismatch d. The force fields F have an energy-momentum and an angular momentum, which provide the conservation law in the space-time geometry of Minkowski.

In general, taking into consideration the force fields F, one may investigate the particle dynamics in the space-time geometry of Minkowski. However, structure of the force fields F is more complicated (function of four points), than the structure of the world function σ_{r} (function of two points). It seems to be more reasonable to investigate the free particle motion in a real (complicated) geometry, than to investigate the particle motion in unknown force fields of the simple space-time geometry $\mathcal{G}_{\mathrm{M}}$.

6 Conclusions

Thus, in the twentieth century a transition from the nonrelativistic physics to the relativistic one has been produced only in dynamic equations, but not in the concept of the particle state. In the nonrelativistic physics the particle state is described as a point in the phase space. Construction of the phase space is founded on the continuous space-time geometry (of Newton, or of Minkowski). Existence of primordially indeterministic particles (elementary particles) is possible only if the space-time geometry is multivariant. One cannot construct phase space, because the limit (1.3), determining the particle momentum, does not exist in a multivariant space-time geometry. We are forced to describe the particle state without limits of the type (1.3).

The relativistic concept of a particle state is realized by means of a skeleton of a particle. The skeleton consists of several space-time points. The number of the skeleton points depends on the structure of the elementary particle. In the simplest case of a pointlike particle its skeleton contains two points. It is important, that the skeleton describes all characteristics of the particle, including its mass, charge, momentum and other characteristics, if they take place, (spin, flower, etc.). As a result one obtains a monistic conception, where all fundamental physical phenomena (including electromagnetic and gravitational interactions) are described in terms of points of the event space and world functions between them.

Dynamic equations are algebraic equations, formulated in a coordinateless form. These equations are simpler and more universal, than equations, used in the conventional theory of elementary particles.

The conventional theory of elementary particles, which uses nonrelativistic concept of particle state, degenerates into phenomenological conception, which systematize elementary particles and their reactions. However, pretenses of the conventional approach to determination of the elementary particles construction are unfounded, because of inconsistent application of relativistic concepts.

A physical theory is a relativistic, if the event space (space-time) is described by one and only one structure: world function σ. If there are another geometric structures, for instance, spatial distant S, the physical theory constructed on such a two-structure geometry of the event space is not relativistic. One can construct a derivative geometric structure – time interval T on the basis of the world function σ and the space distance S. Thereafter one can describe the geometry of the event space on the basis of two structures: space distance S and time interval T, considering them as independent and ignoring the world function σ. Such a two-structure geometry is a Newtonian conception of the event space.

Such a formulation of the difference between the Newtonian and relativistic conceptions of the event space does not refer to the way of transformation of dynamic equations, written in inertial coordinate systems. The invariant (coordinateless) formulation of the relativity theory looks better, than the formulation with a reference to the transformation law of dynamic equations. As we have seen, the reference only to the transformation law and disregard of the relativistic concept of the particle state may lead to an inconsistent conception.

It should distinguish between a conception and a theory. A conception considers connection between different concepts of a theory. For instance, the skeleton conception of elementary particles considers properties of such concepts as skeleton and coordinateless equations of motion. The skeleton conceptions can be used for any choice of world function and of the particle skeleton. The skeleton conception of elementary particles turns to a theory of elementary particles, when the space-time world function has been determined and correspondence between the concrete elementary particles and their skeletons has been established. One can test experimentally the elementary particle theory. However, an experimental test of the skeleton conception is impossible. An experimental test of a conception is meaningless. One can test only a theory constructed on the basis of a conception. Experimental test of a conception looks as an experimental test of the Newton binomial.

References

[1] Yu.A. Rylov, Quantum Mechanics as a theory of relativistic Brownian motion. *Ann. Phys. (Leipzig).* **27**, *1-11, (1971)*

[2] J. E.Moyal, *Proc. Cambr. Phil. Soc.,* **45**, 99, (1949).

[3] I. Fényes, *Zs. f. Phys.*, **132**, 81, (1952).

[4] Yu.A. Rylov, Quantum mechanics as relativistic statistics.I: The two-particle case. *Int. J. Theor. Phys.* **8**, *65-83, (1973).*

[5] Yu.A. Rylov, On quantization of non-linear relativistic field without recourse to perturbation theory. *Int. J. Theor. Phys.* **6**, *181-204. (1972).*

[6] Yu.A. Rylov, Quantum mechanics as relativistic statistics.II: The case of two interacting particles. *Int. J. Theor. Phys.* **8**, *123-139. (1973).*

[7] Yu. A. Rylov, Uniform formalism for description of dynamic quantum and stochastic systems. *e-print /physics/0603237*

[8] Yu.A. Rylov, Spin and wave function as attributes of ideal fluid. *Journ. Math. Phys.* **40**, *pp. 256 - 278, (1999).*

[9] E. Madelung, *Z.Phys.* **40,** *322, (1926).*

[10] D. Bohm, *Phys.Rev.* **85**, *166,(1952), 180,(1952).*

[11] Yu. A. Rylov, Incompatibility of the Copenhagen interpretation with quantum formalism and its reasons. *Concepts of Physics* **5**, *iss.2, 323-328, (2008).* ISSN1897-2357 See also *e-print /physics/0604111*

[12] K. Menger, Untersuchen über allgemeine Metrik, *Mathematische Annalen,* **100**, 75-113, (1928)

[13] L. M. Blumenthal, *Theory and Applications of Distance Geometry*, Oxford, Clarendon Press, 1953.

[14] Yu. A. Rylov, Non-Euclidean method of the generalized geometry construction and its application to space-time geometry. in *Pure and Applied Differential geometry pp.238-246.* eds. Franki Dillen and Ignace Van de Woestyne. Shaker Verlag, Aachen, 2007. (see also *e-print math/0702552.*)

[15] Yu.A. Rylov, Geometry without topology as a new conception of geometry. *Int. Jour. Mat. & Mat. Sci.* **30**, *iss. 12, 733-760, (2002)*, (see also *e-print math.MG/0103002*).

[16] Yu.A. Rylov, Non-Riemannian model of the space-time responsible for quantum effects. *Journ. Math. Phys.* **32(8)**, *2092-2098, (1991)*

[17] Yu. A. Rylov, Generalization of relativistic particle dynamics on the case of non-Riemannian space-time geometry. *Concepts of Physics* **6**, iss.*4, 605, (2009).* ISSN1897-2357. (see also *e-print 0811.4562*).

[18] Yu. A. Rylov, Geometrical dynamics: spin as a result of rotation with superluminal speed *e-print 0801.1913.*

[19] Yu.A. Rylov, Dirac equation in terms of hydrodynamic variables. *Advances in Applied Clifford Algebras,* **5**, *pp 1-40, (1995))* (see also *e-print 1101.5868*)

[20] Yu. A. Rylov, Is the Dirac particle composite? *e-print physics /0410045*

[21] Yu. A. Rylov, Is the Dirac particle completely relativistic? *e-print /physics /0412032.*