Geometric paradigm is a necessity but not a hypothesis

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Abstract

It is shown, that the dominating now quantum paradigm is conditioned by our insufficient knowledge of geometry, when we cannot work with discrete geometries and with geometries, having a limited divisibility. Progress in investigation of geometry admits one to use a more natural and reasonable geometric paradigm, when classical principles of dynamics are fixed, and the space-time geometry is varied.

1 Introduction

There are two different approaches at description of particle dynamics in the microcosm

(1) Quantum paradigm, when the space-time geometry is fixed, but principles of dynamics are varied to explain quantum effects

(2) The geometric paradigm, when the classical principles of dynamics are fixed, but the space-time geometry is varied.

The quantum paradigm, dominating now, has been generated by our insufficient knowledge of geometry, as well as two centuries earlier the axiomatic thermodynamics on the basis of thermogen has been generated by absence of knowledge on molecular structure of matter and by absence of a theory on molecular chaotic motion.

Construction of an effective fundamental physical theory of microcosm is restricted by our poor knowledge of geometry. We can work only with continuous geometries and with infinitely divisible geometries. We cannot work with discrete geometries and with limited divisible geometries. In general, we investigate the methods of the continuous geometry description, supposing that it is a geometry itself. For instance, tie-up of geometrical objects, conditioned by the unlimited divisibility, is declared as confinement and generates an appearance of a fictitious interaction (gluons).

Physical geometry is a science on the shape and mutual dispositions of geometrical objects. At such an approach the geometry is described completely by a distance between all pairs of points in the space or in the space-time. This fact has been known long ago [1, 2]. This fact manifested itself in construction of the metric geometry (metric space), when in the space (on a point set) the distance between all pairs of points is given. In this case the geometry is described completely. Introduction of a coordinate system is not necessary. The dimension of the geometry is also a redundant information, which may disagree with the distance function, because the dimension, (if it can be introduced) is to be obtained from the distance function.

Mathematical geometry is a logical construction on the basis of a system of axioms.

Both geometries: physical and mathematical ones have a common source: the proper Euclidean geometry.

The physical geometry takes the Euclidean geometry in itself from Euclid. It is obtained as a result of a deformation of the proper Euclidean geometry [3, 4]. The physical geometry uses the supposition, that one knows the Euclidean geometry.

The mathematical geometry adopts the method of the proper Euclidean geometry construction with all its problems (necessity of testing the axioms compatibility and necessity of proving numerous theorems). However, the construction of the mathematical geometry does not depend on our knowledge of the proper Euclidean geometry.

The physical geometry is nonaxiomatizable, in general. It means that the physical geometry cannot be deduced from some axiomatics, as a mathematical geometry.

Another problem of the physical geometry lies in the fact, that information which is supplied by the distance function is too abundant. It is not clear, how to use this information effectively. Even if one knows distances between all points of geometrical objects in Fig.1a, one cannot use this information effectively. It is not clear, how one can use a large number of distances between the points of the two objects, although, in principle, their mutual disposition can be described on the basis of this information. However, two spheres in Fig.1b may be described effectively, because description of mutual disposition of the two spheres is carried on by means of three numbers: two radii of spheres and distance between their centers. The difference between the two cases is explained very easily. We are able to describe sphere in terms of distance. However we are not able to describe each of objects in Fig.1a in terms of only distance.

2 Multivariance and axiomatizability

Axiomatizable (mathematical) geometries form a negligible part of all physical geometries, i.e. geometries suitable for description of real space-time. Contem-

Fig.1a Fig1b

porary physicists (and mathematicians) use only axiomatizable geometries. As a result the real space-time geometry remains outside the region of consideration. For instance, one considers only geometry of Minkowski among all uniform isotropic geometries.

The geometry of Minkowski, considered as a physical geometry, is described completely by the world function σ_{M}, The world function

$$\sigma_{\mathrm{M}}(P,Q)=\frac{1}{2}\rho_{\mathrm{M}}^{2}(P,Q)$$

where $\rho_{\mathrm{M}}(P,Q)$ is the distance between points P and Q. In reality there is a lot of uniform isotropic geometries. Each of them is described by the world function $\sigma=F(\sigma_{\mathrm{M}})$, where F is an arbitrary function.

Nonaxiomatizability of a physical geometry is conditioned by its multivariance. A geometry is multivariant with respect to a point P and a vector $\mathbf{AB}$, if at the point P there are many vectors $\mathbf{PQ},\mathbf{PQ}',\mathbf{PQ}'',...$, which are equivalent to the vector $\mathbf{AB}$, but they are not equivalent between themselves. Multivariance of a geometry is connected with its nonaxiomatizability, which in turn is connected with intransitivity of the equivalence relation.

Multivariance of a geometry is a very important property, which was not known before, as well as the concept of inertia was not known in the Aristotelian mechanics

The equivalence relation is transitive in any mathematical geometry, as well as in any logical construction. For this reason the mathematical geometry cannot be multivariant. Deformation of the Euclidean geometry destroys the transitivity of the equivalence relation. The obtained physical geometry secures new properties (multivariance, nonaxiomatizability). The proper Euclidean geometry has not these properties, as well as any mathematical (axiomatizable) geometry.

Construction of a physical geometry is carried out by means of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, which plays a role of a standard geometry. All propositions $\mathcal{P}$ of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ are presented in the form $\mathcal{P}(\sigma_{\mathrm{E}})$, where σ_{E} is the world function of $\mathcal{G}_{\mathrm{E}}$. Thereafter one deforms the standard geometry $\mathcal{G}_{\mathrm{E}}$, replacing σ_{E} by the world function σ of some other physical geometry $\mathcal{G}$: $\mathcal{P}(\sigma_{\mathrm{E}})\rightarrow\mathcal{P}(\sigma)$. One obtains the set $\mathcal{P}(\sigma)$ of all propositions of the physical geometry $\mathcal{G}$. The physical geometry $\mathcal{G}$, obtained from the standard (proper Euclidean) geometry by means of the deformation is not an axiomatizable geometry, in general. Any statement of the new physical geometry associates with some statement of the proper Euclidean geometry. The physical geometry, obtained by means of such a deformation appears to be multvariant and, hence, nonaxiomatizable.

The multivariance is a very important property of the space-time geometry, which is responsible for quantum effects [5].

In the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ the equivalence of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined as follows. Vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent ($\mathbf{P}_0\mathbf{P}_1$eqv $\mathbf{Q}_0\mathbf{Q}_1$), if vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are in parallel ($\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1$) and their lengths $|\mathbf{P}_0\mathbf{P}_1|$ and $|\mathbf{Q}_0\mathbf{Q}_1|$ are equal. Mathematically these two conditions are written in the form

$$(\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1): \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \tag{2.1}$$

$$|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|, \qquad |\mathbf{P}_0\mathbf{P}_1| = \sqrt{2\sigma(P_0, P_1)} \tag{2.2}$$

where $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ is the scalar product of two vectors, defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) \tag{2.3}$$

Here σ is the world function of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. The length $|\mathbf{PQ}|$ of vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{PQ}| = \rho(P, Q) = \sqrt{2\sigma(P, Q)} \tag{2.4}$$

Using relations (2.1) - (2.4), one can write the equivalence condition in the form.

$$\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1: \qquad \sigma(P_0, P_1) = \sigma(Q_0, Q_1)$$

$$\wedge\sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) = \sigma(P_0, P_1) \tag{2.5}$$

The equivalence relation is used in any physical geometry. The definition of equivalence (2.5) is a satisfactory geometrical definition, because it does not contain a reference to a dimension of the space and to a coordinate system. It contains only points P_0, P_1, Q_0, Q_1, determining vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ and distances (world functions) between these points. The definition of equivalence (2.5) coincides with the conventional definition of two vectors equivalence in the proper Euclidean geometry. If in the proper Euclidean geometry one fixes points P_0, P_1, Q_0 in the relations (2.5) and solve these equations with respect to the point Q_1, one finds that these equations always have one and only one solution. This statement follows from the properties of the world function of the proper Euclidean geometry. It means that the proper Euclidean geometry is single-variant with respect any pairs of its points. It means also, that the equivalence relation is transitive in the proper Euclidean geometry. By definition the transitivity of the equivalence relation means, that

$$\text{if} \quad \mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1 \wedge \mathbf{Q}_0\mathbf{Q}_1\text{eqv}\mathbf{R}_0\mathbf{R}_1, \quad \text{then} \quad \mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{R}_0\mathbf{R}_1 \tag{2.6}$$

In the arbitrary physical geometry the equivalence relation has the same form (2.5) with another world function σ, satisfying the constraints

$$\sigma: \quad \Omega \times \Omega \to \mathbb{R}, \qquad \sigma(P, P) = 0, \qquad \forall P, Q \in \Omega \tag{2.7}$$

Here Ω is the set of all points, where the geometry is given.

In the case of arbitrary world function one cannot guarantee, that equations (2.5) have always a unique solution. There may be many solutions. In this case one has a multivariant geometry. There may be no solution. In this case one has a zero-variant (discriminating) geometry. In both cases the equivalence relation is intransitive, and geometry is nonaxiomatizable. There may be such a situation, that the geometry is multivariant with respect to some points and vectors, and it is zero-variant with respect to other points and vectors. Such a geometry will be also qualified as a multivariant geometry.

The geometry of Minkowski is single-variant with respect to timelike vectors, and it is multivariant with respect to spacelike vectors. In the relativistic dynamics the spacelike vectors are not used (superlight velocities are forbidden by the relativity principles). For this reason the multivariance with respect to the spacelike vectors remains to be unknown for physicists.

Let us consider an example of a slightly deformed space-time geometry of Minkowski, given on the manifold of Minkowski. This geometry $\mathcal{G}_{\mathrm{d}}$ is described by the world function σ_{d}

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + \lambda_0^2 \left\{ \begin{array}{ll} \mathrm{sgn}\,(\sigma_{\mathrm{M}}) & \text{if} \quad |\sigma_{\mathrm{M}}| > \sigma_0 \\ \frac{\sigma_{\mathrm{M}}}{\sigma_0} & \text{if} \quad |\sigma_{\mathrm{M}}| \leq \sigma_0 \end{array} \right. , \qquad \lambda_0^2, \sigma_0 = \mathrm{const} \geq 0 \tag{2.8}$$

where σ_{M} is the world function of the geometry $\mathcal{G}_{\mathrm{M}}$ of Minkowski, λ_0 is an elementary length. The world function σ_{M} of the Minkowski geometry $\mathcal{G}_{\mathrm{M}}$ is Lorentz-invariant, and the world function σ_{d} of the granular geometry $\mathcal{G}_{\mathrm{g}}$ is Lorentz-invariant also, because it is a function of σ_{M}. If $\sigma_0 = 0$, the geometry $\mathcal{G}_{\mathrm{d}}$ is discrete, although it is given on the continuous manifold of Minkowski. Indeed, if $\sigma_0 = 0$, in the geometry $\mathcal{G}_{\mathrm{g}}$ there are no close points separated by a distance less, than $\sqrt{2}\lambda_0$. This statement follows from (2.8). *Discrete Lorentz-invariant geometry on a continuous manifold!* This fact seems to be very unexpected at the conventional approach to geometry, where discreteness of geometry depends on the structure of the point set Ω, where the geometry is given, and where the geometry is formulated in some coordinate system.

Granularity (discreteness and continuity at the same time) of the geometry $\mathcal{G}_{\mathrm{d}}$ becomes more clear, if one considers the relative density $\rho\,(\sigma_{\mathrm{d}}) = \frac{d\sigma_{\mathrm{M}}(\sigma_{\mathrm{d}})}{d\sigma_{\mathrm{d}}}$ of points in $\mathcal{G}_{\mathrm{M}}$ with respect to the density of points in $\mathcal{G}_{\mathrm{d}}$. Such a density can be introduced, if both geometries $\mathcal{G}_{\mathrm{d}}$ and $\mathcal{G}_{\mathrm{M}}$ are uniform, and σ_{d} is a function of σ_{M}. One obtains from (2.8)

$$\rho\,(\sigma_{\mathrm{d}}) = \frac{d\sigma_{\mathrm{M}}\,(\sigma_{\mathrm{d}})}{d\sigma_{\mathrm{d}}} = \left\{ \begin{array}{ll} 1 & \text{if} \quad |\sigma_{\mathrm{d}}| > \sigma_0 + \lambda_0^2 \\ \frac{\sigma_0}{\sigma_0 + \lambda_0^2} & \text{if} \quad |\sigma_{\mathrm{d}}| \leq \sigma_0 + \lambda_0^2 \end{array} \right. \tag{2.9}$$

One can see from (2.9), that

$$\begin{array}{rcl} \text{the space-time is continuous } \rho & = & 1, \text{ if } \sigma_{\mathrm{d}} \notin \left(-\sigma_0 - \lambda_0^2, \sigma_0 + \lambda_0^2\right) \\ \text{the space-time is discrete } \rho & = & 0, \text{ if } \sigma_0 = 0 \wedge \sigma_{\mathrm{d}} \in \left(-\lambda_0^2, \lambda_0^2\right) \\ \text{the space-time is granular, if } \sigma_0 & \neq & 0 \end{array}$$

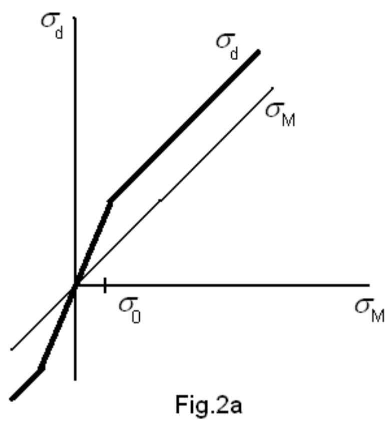


Fig.2a

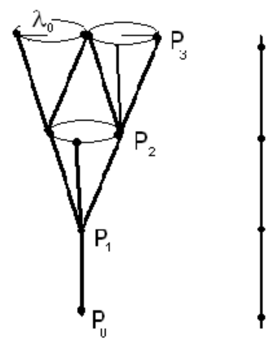


Fig.2b

The intermediate situation, when $\rho = 1$ for large values of $\sigma_{\rm d}$ and $0 \leq \rho < 1$ for small values of $\sigma_{\rm d}$ is treated as a granularity.

In the Fig.2a the function (2.8) is presented. The large values of the world function $\sigma_{\rm d}$ are responsible for multivariance of timelike vectors in the space-time geometry $\mathcal{G}_{\rm d}$. In the Fig.2b we see timelike world lines of pointlike particle in the Minkowski space-time (on the right) and the same world lines in the deformed space-time (on the left). The particle motion in the deformed space-time $\mathcal{G}_{\rm d}$ is multivariant. Statistical description of this multivariant motion leads to quantum description in terms of the Schrödinger equation [5].

Thus, quantum effects are described by means multivariance of the space-time geometry with respect to timelike vectors.

3 Dynamics in physical space-time geometry

In the physical space-time geometry the elementary particle is described by its skeleton $\mathcal{P}_n = \{P_0, P_1, .., P_n\}$, consisting of $n+1$ points. The pointlike particle is described by the skeleton $\mathcal{P}_1 = \{P_0, P_1\}$, consisting of two points, or by the vector $\mathbf{P}_0\mathbf{P}_1$. The vector $\mathbf{P}_0\mathbf{P}_1$ represents the momentum of the pointlike particle, whereas its length $|\mathbf{P}_0\mathbf{P}_1| = \mu$ is the geometrical mass of the pointlike particle. The geometrical mass μ is connected with its usual mass m by means of the relation

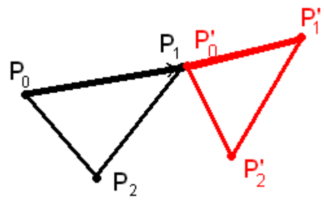
$$m = b\mu \tag{3.1}$$

where b is some universal constant.

Evolution of the elementary particle is described by the world chain, consisting of connected skeletons $...\mathcal{P}_n^{(0)}, \mathcal{P}_n^{(1)}, ..., \mathcal{P}_n^{(s)}...$

$$\mathcal{P}_n^{(s)} = \left\{P_0^{(s)}, P_1^{(s)}, ..P_n^{(s)}\right\}, \qquad s = ...0, 1, ... \tag{3.2}$$

The adjacent skeletons $\mathcal{P}_n^{(s)}, \mathcal{P}_n^{(s+1)}$ of the chain are connected by the relations $P_1^{(s)} = P_0^{(s+1)}$, $s = ...0, 1, ...$ The vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ is the leading vector, which



Number of equations n(n+1), number of variables nN, where n+1 is the number of points in skeleton, N is dimension of the space-time

Fig.3

determined the world chain direction. If the leading vector is timelike, the world chain is timelike. If the leading vector is spacelike, the world chain is spacelike.

Dynamics of free elementary particle is determined by the relations [6]

$$\mathcal{P}_n^{(s)}\text{eqv}\mathcal{P}_n^{(s+1)}: \qquad \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\text{eqv}\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}, \qquad i,k=0,1,..n; \qquad s=...0,1,... \tag{3.3}$$

which describe equivalence of adjacent skeletons.Thus, dynamics of a free elementary particle is described by a system of algebraic equations (3.3). Specific of dynamics depends on the elementary particle structure (number and disposition of particles inside the skeleton) and on the space-time geometry.

In the simplest case, when the space-time geometry is the 5-dimensional Kaluza-Klein geometry [7, 8], the dynamic equations (3.3) for the pointlike particle are reduced to conventional differential dynamic equations, describing motion of the charged pointlike particle in the given electromagnetic and gravitational fields. Thus, dynamic equations (3.3) can be considered as a generalization of classical differential dynamic equations for the particle motion on the case of the granular space-time geometry. It is a very important fact, which shows, that description of free particles by means of a world chain, consisting of connected skeletons, is simply a generalization of conventional relativistic dynamics of particles, which do not interact between themselves. This generalization does not contain any new principles. It is simply a generalization of the particle dynamics onto the case of the granular space-time geometry.

According to definition of dynamics (3.3) all vectors of the skeleton are transported along the chain in parallel with itself (translation), i.e. without a rotation. It means a stronger definition of a free particle, than that, which is used conventionally. Usually the rotating particle, moving in the absence of external fields, is considered to be free, although some parts of the particle move with acceleration, generated by the rotation. In the free motion, defined by the relation (3.3), all points of the skeleton move without an acceleration, and all vectors of the skeleton do not rotate. The particle rotation appears as a special kind of motion with superlight speed (with spacelike leading vector of the world chain). This property seems rather unexpected from conventional viewpoint [9, 10, 11]. However, this property may take place in some special form of the granular geometry. Then the composite particle rotation is realized in the helical shape of the world chain.

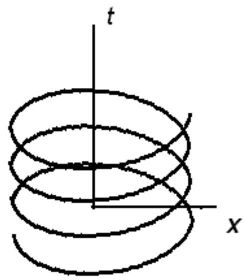


Fig.4a

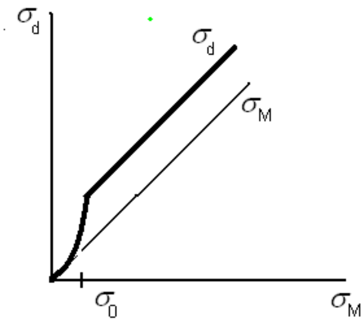


Fig.4b

In the case of timelike world chain the ends of vectors, which are equivalent to a vector, are placed on the proper Euclidean sphere of radius $\sqrt{2}\lambda_0$, as it shown in Fig.2b. As a result the ends of different vectors are placed rather close (at the distance of the order of λ_0). Analogous, in the case of spacelike world chain the ends of equivalent vectors are placed on the sphere of the radius 0 in the pseudo-Euclidean space of index 1. If coordinates of the vector $\mathbf{P}_0\mathbf{P}_1 = \{0, 1, 0, 0\}$, coordinates of equivalent vectors at the point P_0 are $\mathbf{P}_0\mathbf{Q} = \left\{\sqrt{a_2^2 + a_3^2}, 1, a_2, a_3\right\}$, where a_2, a_3 are arbitrary numbers. The sphere of radius 0 in the pseudo-Euclidean space of index 1 is a light cone, whose points may be indefinitely far one from another. It means, that fluctuation of links of the multivariant spacelike world chain may be very large. Such a world chain is not observable. It means, that such a world chain cannot exist.

However, if the world function has the form, shown in Fig.4b, the spacelike world chain may have a shape of a helix with timelike axis. In this case the skeleton is to contain more than, two points and the links of the chain must have rather short lengths [11]. This world chain is shown in Fig.4a. Such a world chain associates with fermion (the Dirac particle) [9, 10]

It is quite reasonable, that the dynamic equations in the granular space-time geometry cannot be differential equations. The dynamic equations can be only difference equations.

4 Incomplete knowledge and false knowledge

Let us imagine an investigator, who knows only axiomatizable geometries and does not know about existence of nonaxiomatizable geometries. He uses the mathematical (axiomatizable) geometries for description of the space-time. The fact, that the investigator does not know physical (nonaxiomatizable) geometries, is not a mistake. It is only incomplete knowledge of geometry. However, if the investigator thinks, that the space-time can be described only in terms of mathematical geometries, and nonaxiomatizable geometries do not exist at all, it becomes to be a mistake. The knowledge of geometry becomes to be a false knowledge. Thus, the false knowledge is not the fact, that we do not know nonaxiomatizable geometries, but the fact, that we do not admit nonaxiomatizable geometries.

Ancient Egyptians believed, that all rivers flow northwards. It was a false knowledge, because ancient Egyptians considered incomplete knowledge (knowledge on one river) as a complete knowledge (knowledge on all rivers), and this self-assurance led to a mistake.

In the same way the fact, we do not know nonaxiomatizable geometries, leads to the quantum paradigm only, if we are very self-confident and believe, that we know geometry very well.

Comprehension of incompleteness of our geometrical knowledge , based on this knowledge, opens the door for progress of our geometrical knowledge and for progress of the microcosm physics, based on this knowledge. Extraneous self-assurance and confidence to completeness and trueness of our geometrical knowledge shut the door for a progress and push us to the path of invention of hypotheses, which compensate incompleteness of our geometrical knowledge. To do it justice the way of compensation may lead to some success, However, finally it leads to blind alley.

5 Concluding remarks

The obtained dynamics is a simple generalization of relativistic dynamics in the Riemannian space-time on the case of the space-time with arbitrary geometry and of a particle with complicated internal structure. This generalization does not use any new ideas and hypotheses. In the case of a pointlike particle and of the Riemannian space-time geometry dynamic equations (3.3) turn into dynamic equations for motion of pointlike particle in the given electromagnetic and gravitational fields. The mass and the charge of the particle are geometrized, i.e. they are some lengths. The particle spin is expressed also via the particle structure (its skeleton). It is very important, that the dynamics of the elementary particle is expressed only in terms of primary (geometrical) quantities. Secondary (derivative) quantities such as wave function, isospin, color an so on do not appear in dynamics. However, these quantities are to be introduced in order a comparison with experiment be possible. Now results of a theory and those of an experiment are formulated in terms of the secondary concepts.

Different elementary particles differ only by its geometrical structure (skeleton).The obtained dynamics is very simple and general.

Generalization of the relativistic dynamics on the case of arbitrary space-time geometry became to be possible due to progress of our geometrical knowledge. This knowledge includes our ability of working with a discrete geometry and with a geometry of unlimited divisibility. Concept of multivariance is an evidence of our progress in geometry and in dynamics, as well as the concept of inertia was an evidence of progress in mechanics in the time of Newton.

This circumstance is formulated in the form: *the geometric paradigm is a neces-*

sity but not a hypothesis. Note that the geometric paradigm is not a new conception. In the end of the nineteenth century the geometric paradigm was a dominating paradigm (without a knowledge of multivariant geometry). Physicists were forced to accept the quantum paradigm (although rather reluctantly).

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