

# Gas dynamics as a tool for description of nondeterministic particles

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## Abstract

Classical gas dynamic equations describe mean motion of stochastic gas molecules. The reason of this stochasticity is interaction (collisions) between molecules. The wave function is the way to describe the gas dynamic equations [3]. If a gas molecules interact via some force field  $\kappa^l$ , the gas dynamic equations have the form of the Klein-Gordon equation provided they are written in terms of the wave function. Among two possible approaches: (i) quantum mechanics (QM) as axiomatic conception and (ii) QM as a kind of gas dynamics the second approach is more preferable, because in the first approach the wave function looks as a strange axiomatic object, whereas in the second approach the wave function is a natural way of the gas dynamics description. Besides the second approach admits one to obtain a more complete description of stochastic particles.

Key words: wave function as attribute of gas dynamics; interaction changing molecular mass; quantum mechanics without quantum principles

## 1 Introduction

Fluid dynamics and, in particular, gas dynamics are considered usually as a tool for calculation of fluid flows. Ludvig Boltzmann had suggested another interpretation of the gas dynamics. He considered the gas dynamic as a tool for statistical description of stochastic gas molecules. At first the pragmatic scientific community did not accept such interpretation of the gas dynamics. However, some time later, the kinetic Boltzmann's equation formed a basis of a mathematical theory of gases [1]. Classical gas dynamic equations describe a mean velocity of gas molecules. Kinetic equation gives a more complete statistical description of stochastic gas molecules.

Conventional statistical description of deterministic particles is produced usually in terms of a statistical ensemble. The statistical ensemble is a set of many independent identical particles. In other words, the statistical ensemble is a collisionless

gas. If the particles of such an ensemble interact between themselves, motion of particles may be stochastic (nondeterministic). Such an ensemble is not considered usually as a statistical ensemble. But ensemble of identical interacting particles can be considered as a statistical ensemble, if the statistical ensemble is considered as a basic object of the particle dynamics (instead of a single particle). See [2] for details. Statistical ensemble with interacting identical particles is a gas. Molecules of this gas move stochastically. The form of the stochasticity depends on the form of the molecular interaction.

It is very important, that elementary particles can be considered as classical nondeterministic (stochastic) particles. Statistical description of these particles is carried out by means of gas dynamics. Such a description is more natural, than conventional quantum description in terms of a wave function, because the wave function is a natural attribute of fluid dynamics [3], whereas the wave function is a strange axiomatic object in the framework of conventional quantum mechanics. This circumstance leads to numerous interpretations of quantum mechanics. These numerous interpretations are conditioned by the fact, that a meaning of the wave function is not clear.

It is well known that the Schrödinger equation can be presented as a description of an irrotational flow of some fluid [4]. D. Bohm developed a connection between the quantum mechanics and dynamics of continuous medium (hydrodynamics) [5]. Unfortunately, the connection between the quantum mechanics and hydrodynamics has been one-sided in the sense, that one could obtain the hydrodynamic description from quantum mechanics, but one cannot obtain the Schrödinger equation from hydrodynamical equations. The recent papers by E.Nelson [6] did not change this situation, although Nelson tried to consider classical stochastic particles. The reason of such a situation is the fact, that the wave function is an axiomatic object in quantum mechanics, and nobody does not know, what is a wave function.

Situation changed, when it became known, that the wave function is a description method of any nondissipative continuous medium [3]. There are three methods of the gas dynamics equations presentation: (1) Euler presentation, (2) Lagrangian presentation and (3) presentation in terms of wave function. The last presentation was not known in the twentieth century. It became to be known only in the end of the XX century [3]. As far as the wave function is a natural attribute of the continuous medium dynamics, it seems to be more reasonable to consider classical dynamics of fluids as a primary conception of quantum mechanics. In quantum mechanics the wave function is a strange axiomatic object. Then the quantum mechanics will be a secondary (derivative) conception, describing quantum effects.

*The classical gas dynamics can be considered as a natural method of a stochastic particle description.* Indeed, the gas molecule moves stochastically due to interaction with other gas molecules. This interaction reveals because of molecular collisions. If there are no collisions, the gas molecules move deterministically. The character of stochasticity depends on the form of molecular interaction. It turns out that one can introduce such a molecular interaction, when the irrotational flow of the gas will be described by the Klein - Gordon equation, if such an interaction between

molecules takes place. This interaction changes the molecular mass  $m$ , converting it into the effective mass  $M$  by means of the relation

$$m^2 \rightarrow M^2(x) = m^2 + \frac{\hbar^2}{c^2} (g_{kl}\kappa^k\kappa^l + \partial_l\kappa^l), \quad \partial_l \equiv \frac{\partial}{\partial x^l} \quad (1.1)$$

where  $\kappa^l$ ,  $l = 0, 1, 2, 3$  is some force field and  $\hbar$  is the quantum constant. Dynamic equations for the  $\kappa$ -field are obtained from the corresponding action. From these dynamic equations it follows that the  $\kappa$ -field has the potential  $\kappa$

$$\kappa_l = g_{lk}\kappa^k = \partial_l\kappa, \quad l = 0, 1, 2, 3 \quad (1.2)$$

In a usual gas the interaction between molecules is actual in the following sense. A motion of a single molecule (outside the gas) becomes to be deterministic. If the gas dynamics is used as a method of description of the stochastic (quantum) particles motion, the interaction of particles is fictitious in the sense that the motion of a single particle remains to be stochastic.

The classical gas dynamics describes a mean motion of a molecule. This description is rather incomplete in the sense, that it describes only mean values  $\langle \mathbf{p} \rangle$ ,  $\langle E \rangle$  and  $\langle \mathbf{p} \times \mathbf{x} \rangle$  of only additive quantities such as momentum  $\mathbf{p}$ , energy  $E$  and angular momentum  $\mathbf{p} \times \mathbf{x}$ . It is connected with the fact that the classical gas dynamic equations have been deduced from the conservation laws of matter and of momentum. In that time the molecular structure of the gas was not known. The gas dynamic equations cannot describe the fluctuation of the molecule energy  $\left\langle \sqrt{\langle E^2 \rangle - \langle E \rangle^2} \right\rangle$ . To obtain higher moments of  $\mathbf{p}$ , one needs to use the distribution function  $f(\mathbf{x}, \mathbf{p})$ . It can be obtained from the kinetic equation, which takes into account more detailed information on the molecules interaction (collisions). Ludvig Boltzmann investigated interaction between molecules (collisions). As a result he succeeded to obtain kinetic equation describing evolution of the distribution function  $f(\mathbf{x}, \mathbf{p})$ . The distribution function describes completely the molecular stochasticity.

If formalism of the gas dynamics is used for description of the mean motion of quantum particles, it admits one to describe only mean quantities  $\langle \mathbf{p} \rangle$ ,  $\langle E \rangle$  and  $\langle \mathbf{p} \times \mathbf{x} \rangle$ . It is incomplete description. To obtain a more detailed information on the mean motion of a quantum particle, one needs to use a more detailed information on the  $\kappa$ -field. The distribution function  $f(\mathbf{x}, \mathbf{p})$  is a nonrelativistic structure, as well as the phase space, where the distribution function is defined. The quantum particle is a relativistic particle. The regular component of velocity may be nonrelativistic, whereas the stochastic component of velocity is relativistic. This circumstance is important in the relation, that the formalism of quantum mechanics, based on the wave function formalism, gives only incomplete description of mean motion of a quantum particle. However, the axiomatic quantum mechanics claims that the description in terms of wave function is maximally complete. As a result the elementary particles are described as a pointlike objects, provided with various quantum numbers. A possible internal structure of elementary particles remains to be unknown.

The gas, whose molecules interact via the  $\kappa$ -field (1.1), is described by the action

$$\mathcal{E}[S_{\text{st}}]: \quad \mathcal{A}[x, \kappa] = \int_{\xi_0} \int_{V_\xi} \left( -mcK \sqrt{g_{lk} \dot{x}^l \dot{x}^k} - \frac{e}{c} A_l \dot{x}^l \right) d^4 \xi, \quad \dot{x}^i = \frac{\partial x^i}{\partial \xi_0} \quad (1.3)$$

$$K = \frac{M}{m} = \sqrt{1 + \lambda^2 (\kappa_l \kappa^l + \partial_l \kappa^l)}, \quad \lambda = \frac{\hbar}{mc}, \quad \partial_l \equiv \frac{\partial}{\partial x^l} \quad (1.4)$$

where  $\xi = \{\xi_0, \boldsymbol{\xi}\}$ ,  $A_l$  is electromagnetic potential and  $e$  is the molecule charge. The variables  $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$  label world lines of molecules, whereas  $\xi_0$  is a parameter along the world line. The motion of gas molecules is stochastic one. Indeed, the action of a single gas molecule is written in the form (integration over  $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$  is omitted)

$$S_{\text{st}}: \quad \mathcal{A}[x, \kappa] = \int_{\xi_0} \left( -mcK \sqrt{g_{lk} \dot{x}^l \dot{x}^k} - \frac{e}{c} A_l \dot{x}^l \right) d\xi_0 \quad \dot{x}^i = \frac{\partial x^i}{\partial \xi_0}, \quad (1.5)$$

If  $K$  is defined by (1.4) and  $\kappa^l$  does not vanish, the action (1.5) is defined incorrectly, because  $x^k = x^k(\xi_0)$  in (1.5) is one-dimensional line, whereas derivatives of  $\kappa^l$  in  $K$  are defined in the whole space-time. One cannot obtain dynamic equation from the action (1.5). This means that there are no dynamic equations for a single molecule. Hence, molecules of the gas are stochastic. However, if there are no interaction between molecules and  $\kappa^l \equiv 0$ , then  $K \equiv 1$ . In this case the action (1.5) generates dynamic equations for a single molecule, and the molecule motion is deterministic.

Note, that not any interaction between the gas molecules generates their stochasticity. For instance, electromagnetic interaction between charged molecules of a gas does not generate stochasticity. In this case instead of (1.3) we have

$$\mathcal{E}[S_{\text{d}}]: \quad \mathcal{A}[x] = \int_{\xi_0} \int_{V_\xi} \left( -mc \sqrt{g_{lk} \dot{x}^l \dot{x}^k} - \frac{e}{c} A_l \dot{x}^l \right) d^4 \xi - \frac{1}{16\pi c} \int_{V_x} F_{ik} F^{ik} d^4 x \quad (1.6)$$

$$F_{ik} = \partial_i A_k - \partial_k A_i \quad (1.7)$$

The action for a single molecule is obtained from (1.6) in the form

$$S_{\text{d}}: \quad \mathcal{A}[x] = \int_{\xi_0} \left( -mc \sqrt{g_{lk} \dot{x}^l \dot{x}^k} - \frac{e}{c} A_l \dot{x}^l \right) d\xi_0 - \frac{1}{16\pi c} \int_{V_x} F_{ik} F^{ik} d^4 x \quad (1.8)$$

Action (1.8) is defined correctly. As a result action (1.8) generates dynamic equations for a single particle. Thus, electromagnetic interaction does not generate stochasticity of molecules in the gas, consisted of charged molecules.

## 2 Wave function as a method of the gas description

The complete system of the gas dynamic equations includes seven equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla p(\rho)}{\rho} \quad (2.1)$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(t, \mathbf{x}) \quad (2.2)$$

where  $\rho$  is the density,  $\mathbf{v}$  is the velocity and  $p$  is the pressure in the gas. Equations (2.1) form a closed subsystem of the whole system (2.1), (2.2). Ordinary differential equations (2.2) describe a motion of gas particles in a given velocity field. Let  $\boldsymbol{\xi}(t, \mathbf{x}) = (\xi_1(t, \mathbf{x}), \xi_2(t, \mathbf{x}), \xi_3(t, \mathbf{x}))$  be three independent integrals of equations (2.2). Then three equations

$$\frac{\partial \boldsymbol{\xi}}{\partial t} + (\mathbf{v} \nabla) \boldsymbol{\xi} = 0 \quad (2.3)$$

are equivalent to equations (2.2). The dynamic system of seven equations (2.1), (2.3) is equivalent to the system of equations (2.1), (2.2). Equations (2.3) are ordinary differential equations, although they have the form of partial differential equations.

The system of equations (2.1), (2.3) can be integrated in the form

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0, \quad \mathbf{p} = b_0(\nabla \varphi + g^\alpha(\boldsymbol{\xi}) \nabla \xi_\alpha) \quad (2.4)$$

$$\frac{\partial \boldsymbol{\xi}}{\partial t} + (\mathbf{v} \nabla) \boldsymbol{\xi} = 0 \quad (2.5)$$

where  $b_0$  is an arbitrary constant,  $\varphi$  is a dynamical variable, appeared instead of fictitious variable  $\xi_0$ ,  $g^\alpha(\boldsymbol{\xi})$ ,  $\alpha = 1, 2, 3$  are three arbitrary functions of  $\boldsymbol{\xi}$ , and  $\mathbf{p} = \{p_1, p_2, p_3\}$  is the momentum, which is connected with the velocity  $\mathbf{v}$  by the relation

$$p_\alpha = \frac{mv^\alpha}{\sqrt{1 - c^{-2}\mathbf{v}^2}}, \quad \text{or} \quad p_\alpha = mv^\alpha, \quad \alpha = 1, 2, 3 \quad (2.6)$$

Details of this integration can be found in [7]. The first relation (2.6) takes place in the relativistic case. The second one takes place in the non-relativistic case. Arbitrary functions  $g^\alpha(\boldsymbol{\xi})$ ,  $\alpha = 1, 2, 3$  can be expressed via initial values of momentum  $\mathbf{p}$ . Indeed, if we put  $t = \xi_0 = 0$ ,  $\boldsymbol{\xi} = \mathbf{x}$ ,  $\varphi(\mathbf{x}) = 0$ , the second Eq.(2.4) takes the form

$$p_\alpha(0, \mathbf{x}) = b_0 g^\alpha(\mathbf{x}), \quad \alpha = 1, 2, 3 \quad (2.7)$$

Integration (2.4) was produced by Clebsch for incompressible fluid [8, 9]. Integration (2.4) does not depend on the form of the internal energy of the gas, or on the form of the pressure  $p(\rho)$ .

Let us note that the close subsystem of four equations (2.1) cannot be integrated in the form (2.4). It is a reason, why the Clebsch potentials (2.4) have not be used for a long time. Most researchers considered the closed subsystem (2.1) as a complete system of gas dynamic equations on that reason that the system (2.1) is a closed system of differential equations.

The wave function  $\psi = \{\psi_\alpha\}$ ,  $\alpha = 1, 2, \dots, n$  is a  $n$ -component complex function. It is constructed of Clebsch potentials by means of relations

$$\psi_\alpha = \sqrt{\rho} e^{i\varphi} w_\alpha(\boldsymbol{\xi}), \quad \psi_\alpha^* = \sqrt{\rho} e^{-i\varphi} w_\alpha^*(\boldsymbol{\xi}), \quad \alpha = 1, 2, \dots, n, \quad (2.8)$$

$$\psi^* \psi \equiv \sum_{\alpha=1}^n \psi_{\alpha}^* \psi_{\alpha}, \quad (2.9)$$

where (\*) means the complex conjugate,  $w_{\alpha}(\boldsymbol{\xi})$ ,  $\alpha = 1, 2, \dots, n$  are functions of only variables  $\boldsymbol{\xi}$ . They satisfy the relations

$$-\frac{i}{2} \sum_{\alpha=1}^n \left( w_{\alpha}^* \frac{\partial w_{\alpha}}{\partial \xi_{\beta}} - \frac{\partial w_{\alpha}^*}{\partial \xi_{\beta}} w_{\alpha} \right) = g^{\beta}(\boldsymbol{\xi}), \quad \beta = 1, 2, 3, \quad \sum_{\alpha=1}^n w_{\alpha}^* w_{\alpha} = 1. \quad (2.10)$$

The number  $n$  is such a natural number, that equations (2.10) admit a solution. In general,  $n$  may depend on the form of the arbitrary integration functions  $\mathbf{g} = \{g^{\beta}(\boldsymbol{\xi})\}$ ,  $\beta = 1, 2, 3$ . Practically  $n = 1$  for irrotational flow, when  $\mathbf{g}(\boldsymbol{\xi}) = 0$ . In the case of vortical flow  $n = 2$ .

Thus, the wave function is a method of the continuous medium description.

### 3 Transformation of the gas dynamic equations to the form of description in terms of wave function

Let us transform action (1.3) and (1.4) to the description in terms of the wave function. We consider variables  $\xi = \xi(x)$  in (1.3) as dependent variables and variables  $x$  as independent variables. Let the relevant transformation Jacobian

$$J = \frac{\partial(\xi_0, \xi_1, \xi_2, \xi_3)}{\partial(x^0, x^1, x^2, x^3)} = \det \|\xi_{i,k}\|, \quad \xi_{i,k} \equiv \partial_k \xi_i \equiv \frac{\partial \xi_i}{\partial x^k}, \quad i, k = 0, 1, 2, 3 \quad (3.1)$$

be considered to be a multilinear function of  $\xi_{i,k}$ . Then

$$d^4 \xi = J d^4 x, \quad \dot{x}^i \equiv \frac{dx^i}{d\xi_0} \equiv \frac{\partial(x^i, \xi_1, \xi_2, \xi_3)}{\partial(\xi_0, \xi_1, \xi_2, \xi_3)} = J^{-1} \frac{\partial J}{\partial \xi_{0,i}} \quad (3.2)$$

After transformation to dependent variables  $\xi$  action (1.3) takes the form

$$\mathcal{A}[\xi, \kappa] = \int \left\{ -mcK \sqrt{g_{ik} \frac{\partial J}{\partial \xi_{0,i}} \frac{\partial J}{\partial \xi_{0,k}}} - \frac{e}{c} A_k \frac{\partial J}{\partial \xi_{0,k}} \right\} d^4 x, \quad (3.3)$$

$$K = \sqrt{1 + \lambda^2 (\kappa_l \kappa^l + \partial_l \kappa^l)}, \quad \lambda = \frac{\hbar}{mc}, \quad (3.4)$$

Now variables  $\xi$  and  $\kappa$  are considered as functions of independent variables  $x = \{x^0, x^1, x^2, x^3\}$ .

Let us introduce new variables

$$j^k = \frac{\partial J}{\partial \xi_{0,k}}, \quad k = 0, 1, 2, 3 \quad (3.5)$$

by means of Lagrange multipliers  $p_k$

$$\mathcal{A}[\xi, \kappa, j, p] = \int \left\{ -mcK \sqrt{g_{ik}j^i j^k} - \frac{e}{c} A_k j^k + p_k \left( \frac{\partial J}{\partial \xi_{0,k}} - j^k \right) \right\} d^4x \quad (3.6)$$

Variation with respect to  $\xi_i$  gives

$$\frac{\delta \mathcal{A}}{\delta \xi_i} = -\partial_l \left( p_k \frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \right) = 0, \quad i = 0, 1, 2, 3 \quad (3.7)$$

Using the identities

$$\frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \equiv J^{-1} \left( \frac{\partial J}{\partial \xi_{0,k}} \frac{\partial J}{\partial \xi_{i,l}} - \frac{\partial J}{\partial \xi_{0,l}} \frac{\partial J}{\partial \xi_{i,k}} \right) \quad (3.8)$$

$$\frac{\partial J}{\partial \xi_{i,l}} \xi_{k,l} \equiv J \delta_k^i, \quad \partial_l \frac{\partial J}{\partial \xi_{i,l}} \equiv 0 \quad \partial_l \frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \equiv 0 \quad (3.9)$$

one can test by direct substitution that the general solution of linear equations (3.7) has the form

$$p_k = b_0 (\partial_k \varphi + g^\alpha(\boldsymbol{\xi}) \partial_k \xi_\alpha), \quad k = 0, 1, 2, 3 \quad (3.10)$$

where  $b_0 \neq 0$  is a constant,  $g^\alpha(\boldsymbol{\xi})$ ,  $\alpha = 1, 2, 3$  are arbitrary functions of  $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$ , and  $\varphi$  is the dynamic variable  $\xi_0$ , which ceases to be fictitious. Let us substitute (3.10) in (3.6). The term of the form  $\partial_k \varphi \partial J / \partial \xi_{0,k}$  is reduced to Jacobian and does not contribute to dynamic equations. The terms of the form  $\xi_{\alpha,k} \partial J / \partial \xi_{0,k}$  vanish due to identities (3.9). We obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa, j] = \int \left\{ -mcK \sqrt{g_{ik}j^i j^k} - j^k \pi_k \right\} d^4x, \quad (3.11)$$

where quantities  $\pi_k$  are determined by the relations

$$\pi_k = b_0 (\partial_k \varphi + g^\alpha(\boldsymbol{\xi}) \partial_k \xi_\alpha) + \frac{e}{c} A_k, \quad k = 0, 1, 2, 3 \quad (3.12)$$

Integration of (3.7) in the form (3.10) is such an integration that allows to introduce a wave function. Note that coefficients in the system of equations (3.7) at derivatives of  $p_k$  are constructed of minors of the Jacobian (3.1). It is the circumstance that enables one to carry out a formal general integration.

Variation of (3.11) with respect to  $\kappa^l$  gives

$$\frac{\delta \mathcal{A}}{\delta \kappa^l} = -\frac{\lambda^2 mc \sqrt{g_{ik}j^i j^k}}{K} \kappa_l + \partial_l \frac{\lambda^2 mc \sqrt{g_{ik}j^i j^k}}{2K} = 0, \quad \lambda = \frac{\hbar}{mc} \quad (3.13)$$

It can be written in the form

$$\kappa_l = \partial_l \kappa = \frac{1}{2} \partial_l \ln \rho, \quad \rho = \frac{\sqrt{j_s j^s}}{K} \quad (3.14)$$

or

$$e^{2\kappa} = \frac{\rho}{\rho_0} \equiv \frac{\sqrt{j_s j^s}}{\rho_0 K} \quad (3.15)$$

where the variable  $\kappa$  is a potential of the  $\kappa$ -field  $\kappa_i$  and  $\rho_0 = \text{const}$  is an integration constant. Substituting (3.4) in (3.15), we obtain dynamic equation for  $\kappa$

$$\hbar^2 (\partial_l \kappa \cdot \partial^l \kappa + \partial_l \partial^l \kappa) = m^2 c^2 \frac{e^{-4\kappa} j_s j^s}{\rho_0^2} - m^2 c^2 \quad (3.16)$$

Variation of (3.11) with respect to  $j^k$  gives

$$\pi_k = -\frac{mcKj_k}{\sqrt{g_{ls}j^l j^s}} \quad (3.17)$$

or

$$\pi_k g^{kl} \pi_l = m^2 c^2 K^2 \quad (3.18)$$

Substituting  $\sqrt{j_s j^s}/K$  from the second Eq. (3.14) in (3.17), we obtain

$$j_k = -\frac{\rho_0}{mc} e^{2\kappa} \pi_k, \quad (3.19)$$

Now we eliminate variables  $j^k$  from the action (3.11), using relation (3.19) and (3.15). We obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa] = \int \rho_0 e^{2\kappa} \{-m^2 c^2 K^2 + \pi^k \pi_k\} d^4 x, \quad (3.20)$$

where  $\pi_k$  is determined by the relation (3.12). Using expression (1.4) and (3.15) for  $K$ , the first term of the action (3.20) can be transformed as follows.

$$\begin{aligned} -m^2 c^2 e^{2\kappa} K^2 &= -m^2 c^2 e^{2\kappa} (1 + \lambda^2 (\partial_l \kappa \partial^l \kappa + \partial_l \partial^l \kappa)) \\ &= -m^2 c^2 e^{2\kappa} + \hbar^2 e^{2\kappa} \partial_l \kappa \partial^l \kappa - \frac{\hbar^2}{2} \partial_l \partial^l e^{2\kappa} \end{aligned}$$

Let us take into account that the last term has the form of divergence. It does not contribute to dynamic equations and can be omitted. After omitting this term, we obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa] = \int \rho_0 e^{2\kappa} \{-m^2 c^2 + \hbar^2 \partial_l \kappa \partial^l \kappa + \pi^k \pi_k\} d^4 x, \quad (3.21)$$

Here  $\pi_k$  is defined by the relation (3.12), where the integration constant  $b_0$  is chosen in the form  $b_0 = \hbar$

$$\pi_k = \hbar (\partial_k \varphi + g^\alpha (\boldsymbol{\xi}) \partial_k \xi_\alpha) + \frac{e}{c} A_k, \quad k = 0, 1, 2, 3 \quad (3.22)$$

Instead of dynamic variables  $\varphi, \boldsymbol{\xi}, \kappa$  we introduce two-component complex function (2.8) and (2.9)

$$\psi = \{\psi_\alpha\} = \{\sqrt{\rho} e^{i\varphi} w_\alpha(\boldsymbol{\xi})\} = \{\sqrt{\rho_0} e^{\kappa+i\varphi} w_\alpha(\boldsymbol{\xi})\}, \quad \alpha = 1, 2 \quad (3.23)$$



Here  $w_\alpha$  are functions of only  $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$ , having the following properties

$$\sum_{\alpha=1}^{\alpha=2} w_\alpha^* w_\alpha = 1, \quad -\frac{i}{2} \sum_{\alpha=1}^{\alpha=2} \left( w_\alpha^* \frac{\partial w_\alpha}{\partial \xi_\beta} - \frac{\partial w_\alpha^*}{\partial \xi_\beta} w_\alpha \right) = g^\beta(\boldsymbol{\xi}) \quad (3.24)$$

where  $(*)$  denotes the complex conjugation. The number of components of the wave function  $\psi$  depends on the functions  $g^\beta(\boldsymbol{\xi})$ . This number is chosen in such a way, that equations (3.24) have a solution. Then we obtain

$$\psi^* \psi \equiv \sum_{\alpha=1}^{\alpha=2} \psi_\alpha^* \psi_\alpha = \rho = \rho_0 e^{2\kappa}, \quad \partial_l \kappa = \frac{\partial_l (\psi^* \psi)}{2\psi^* \psi} \quad (3.25)$$

$$\pi_k = -\frac{i\hbar (\psi^* \partial_k \psi - \partial_k \psi^* \cdot \psi)}{2\psi^* \psi} + \frac{e}{c} A_k, \quad k = 0, 1, 2, 3 \quad (3.26)$$

Substituting relations (3.25), (3.26) in (3.21), we obtain the action, written in terms of the wave function  $\psi$

$$\begin{aligned} \mathcal{A}[\psi, \psi^*] = & \int \left\{ \left[ \frac{i\hbar (\psi^* \partial_k \psi - \partial_k \psi^* \cdot \psi)}{2\psi^* \psi} - \frac{e}{c} A_k \right] \left[ \frac{i\hbar (\psi^* \partial^k \psi - \partial^k \psi^* \cdot \psi)}{2\psi^* \psi} - \frac{e}{c} A^k \right] \right. \\ & \left. + \hbar^2 \frac{\partial_l (\psi^* \psi) \partial^l (\psi^* \psi)}{4(\psi^* \psi)^2} - m^2 c^2 \right\} \psi^* \psi d^4 x \end{aligned} \quad (3.27)$$

Let us use the identity

$$\begin{aligned} & \frac{(\psi^* \partial_l \psi - \partial_l \psi^* \cdot \psi) (\psi^* \partial^l \psi - \partial^l \psi^* \cdot \psi)}{4\psi^* \psi} + \partial_l \psi^* \partial^l \psi \\ \equiv & \frac{\partial_l (\psi^* \psi) \partial^l (\psi^* \psi)}{4\psi^* \psi} + \frac{g^{ls}}{2} \psi^* \psi \sum_{\alpha, \beta=1}^{\alpha, \beta=2} Q_{\alpha\beta, l}^* Q_{\alpha\beta, s} \end{aligned} \quad (3.28)$$

where

$$Q_{\alpha\beta, l} = \frac{1}{\psi^* \psi} \begin{vmatrix} \psi_\alpha & \psi_\beta \\ \partial_l \psi_\alpha & \partial_l \psi_\beta \end{vmatrix}, \quad Q_{\alpha\beta, l}^* = \frac{1}{\psi^* \psi} \begin{vmatrix} \psi_\alpha^* & \psi_\beta^* \\ \partial_l \psi_\alpha^* & \partial_l \psi_\beta^* \end{vmatrix} \quad (3.29)$$

Then we obtain

$$\begin{aligned} \mathcal{A}[\psi, \psi^*] = & \int \left\{ \left( i\hbar \partial_k + \frac{e}{c} A_k \right) \psi^* \left( -i\hbar \partial^k + \frac{e}{c} A^k \right) \psi - m^2 c^2 \psi^* \psi \right. \\ & \left. + \frac{\hbar^2}{2} \sum_{\alpha, \beta=1}^{\alpha, \beta=n} g^{ls} Q_{\alpha\beta, l}^* Q_{\alpha\beta, s} \psi^* \psi \right\} d^4 x \end{aligned} \quad (3.30)$$

Let us consider the case of irrotational flow, when  $g^\alpha(\boldsymbol{\xi}) = 0$ . In this case  $w_1 = 1$ ,  $w_2 = 0$ , and the function  $\psi$  has only one component. It follows from (3.29), that  $Q_{\alpha\beta, l} = 0$ . Then instead of (3.30) we obtain

$$\mathcal{A}[\psi, \psi^*] = \int \left\{ \left( i\hbar \partial_k + \frac{e}{c} A_k \right) \psi^* \left( -i\hbar \partial^k + \frac{e}{c} A^k \right) \psi - m^2 c^2 \psi^* \psi \right\} d^4 x \quad (3.31)$$

Variation of action (3.31) with respect to  $\psi^*$  generates the Klein - Gordon equation

$$\left(-i\hbar\partial_k + \frac{e}{c}A_k\right) \left(-i\hbar\partial^k + \frac{e}{c}A^k\right) \psi - m^2c^2\psi = 0 \quad (3.32)$$

Thus, description in terms of the Klein - Gordon equation is a special case of the stochastic particles description by means of the action (1.3), (1.4).

In the case, when the fluid flow is rotational, and the wave function  $\psi$  is two-component, identity (3.28) takes the form

$$\begin{aligned} & \frac{(\psi^*\partial_l\psi - \partial_l\psi^* \cdot \psi)(\psi^*\partial^l\psi - \partial^l\psi^* \cdot \psi)}{4\rho} - \frac{(\partial_l\rho)(\partial^l\rho)}{4\rho} \\ & \equiv -\partial_l\psi^*\partial^l\psi + \frac{1}{4}(\partial_l s_\alpha)(\partial^l s_\alpha)\rho \end{aligned} \quad (3.33)$$

where 3-vector  $\mathbf{s} = \{s_1, s_2, s_3, \}$  is defined by the relations

$$\rho = \psi^*\psi, \quad s_\alpha = \frac{\psi^*\sigma_\alpha\psi}{\rho}, \quad \alpha = 1, 2, 3 \quad (3.34)$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \psi^* = (\psi_1^*, \psi_2^*) \quad (3.35)$$

Here  $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$  are Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3.36)$$

Note that 3-vectors  $\mathbf{s}$  and  $\boldsymbol{\sigma}$  are vectors in the space  $V_\xi$  of the Lagrange coordinates  $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$ . They are transformed as vectors under the transformations

$$\xi_\alpha \rightarrow \tilde{\xi}_\alpha = \tilde{\xi}_\alpha(\boldsymbol{\xi}), \quad \alpha = 1, 2, 3, \quad \frac{\partial(\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3)}{\partial(\xi_1, \xi_2, \xi_3)} \neq 0 \quad (3.37)$$

In general, transformations of the Lagrange coordinates  $\boldsymbol{\xi}$  and those of coordinates  $\mathbf{x}$  are independent. However, action (3.27) does not contain any reference to the Lagrange coordinates  $\boldsymbol{\xi}$  and transformations (3.37) of  $\boldsymbol{\xi}$ . If we consider only linear transformations of space coordinates  $\mathbf{x}$

$$x^\alpha \rightarrow \tilde{x}^\alpha = b^\alpha + \omega_{\beta}^{\alpha}x^\beta, \quad \alpha = 1, 2, 3 \quad (3.38)$$

nothing prevents from accompanying any transformation (3.38) with the similar transformation

$$\xi_\alpha \rightarrow \tilde{\xi}_\alpha = b^\alpha + \omega_{\beta}^{\alpha}\xi_\beta, \quad \alpha = 1, 2, 3 \quad (3.39)$$

of Lagrange coordinates  $\boldsymbol{\xi}$ . The formulas for linear transformation of vectors and spinors in  $V_x$  do not contain the coordinates  $\mathbf{x}$  explicitly, and one can consider vectors and spinors in  $V_\xi$  as vectors and spinors in  $V_x$ , provided we always consider linear transformations (3.38) and (3.39) together.

Using identity (3.33), we obtain from (3.27)

$$\mathcal{A}[\psi, \psi^*] = \int \left\{ \left( i\hbar\partial_k + \frac{e}{c}A_k \right) \psi^* \left( -i\hbar\partial^k + \frac{e}{c}A^k \right) \psi - m^2c^2\rho - \frac{\hbar^2}{4} (\partial_l s_\alpha) (\partial^l s_\alpha) \rho \right\} d^4x \quad (3.40)$$

Dynamic equation, generated by action (3.40), has the form

$$\begin{aligned} & \left( -i\hbar\partial_k + \frac{e}{c}A_k \right) \left( -i\hbar\partial^k + \frac{e}{c}A^k \right) \psi - \left( m^2c^2 + \frac{\hbar^2}{4} (\partial_l s_\alpha) (\partial^l s_\alpha) \right) \psi \\ &= -\hbar^2 \frac{\partial_l (\rho \partial^l s_\alpha)}{2\rho} (\sigma_\alpha - s_\alpha) \psi \end{aligned} \quad (3.41)$$

The gradient of the unit 3-vector  $\mathbf{s} = \{s_1, s_2, s_3\}$  describes rotational component of the gas flow. If  $\mathbf{s} = \text{const}$ , the dynamic Eq. (3.41) converts into the conventional Klein - Gordon Eq. (3.32).

## 4 Interrelation between the axiomatic conception and the gas dynamical conception of quantum mechanics.

Axiomatic conception (AC) of quantum mechanics (QM) and the gas dynamic conception (GDC) of QM lead to the same dynamic equations, and one may speak that mathematical formalism of CGC of QM is equivalent to the mathematical formalism of conventional QM. However, the AC of QM contains the wave function, which is an axiomatic object. The meaning of the wave function is not clear and there are necessary additional interpretation of quantum mechanics. There are several various interpretations of QM, and some of them are incompatible with the mathematical formalism of the quantum mechanics. Interpretation of any classical dynamic system  $\mathcal{S}$  is realized in terms of dynamical variables of the system  $\mathcal{S}$ , and additional interpretation of gas dynamic conception is not necessary.

The Copenhagen interpretation is the most widespread interpretation of quantum mechanics. In this interpretation one supposes that wave function describes a state of a single particle. Unfortunately, this supposition is incompatible with mathematical formalism of quantum mechanics. Indeed, let us write the Schrödinger equation in terms of two real equations for wave function  $\psi = \sqrt{\rho} \exp(S/i)$ . One obtains

$$\partial_t \rho + \frac{1}{m} \nabla (\rho \nabla S) = 0 \quad (4.1)$$

$$\partial_t S - \frac{1}{2m} (\nabla S)^2 + \frac{\hbar^2}{2m\sqrt{\rho}} \nabla^2 \sqrt{\rho} = 0 \quad (4.2)$$

If one sets  $\hbar = 0$  in (4.2), one obtains the Jacoby - Hamilton equation for a free classical particle.

$$\partial_t S - \frac{1}{2m} (\nabla S)^2 = 0 \quad (4.3)$$

It means that equations (4.1) and (4.3) describe statistical ensemble of classical particles. It means that the Schrödinger equation describes a statistical ensemble of quantum particles. It cannot describe a single quantum particle, because in this case the classical limit  $\hbar \rightarrow 0$  leads to a single classical particle, but not to statistical ensemble of classical particles. Thus, in reality the wave function describes a state of a statistical ensemble quantum particles. The same conclusion follows from the action (1.3).

These two different interpretations differ in the relation, that in the case of statistical ensemble one has two different kinds of measurement, whereas in the case of a single particle one has only one kind of measurement. For the statistical ensemble there are (1) massive measurement (M-measurement) that is produced over all particles of the statistical ensemble, and (2) single measurement (S-measurement) that is produced only over one of the ensemble particles. S-measurement and M-measurement have different properties, and they cannot be confused.

In the case, when the wave function describes a state of a single particle, there is only S-measurement. One obtains paradoxes, when the single measurement is used instead of M-measurement. It is essential in the case, when a measurement (M-measurement) changes the wave function (reduction of the wave function as a result of M-measurement in process of entanglement and so on). We shall not consider details of these phenomena here. They can be found in [10, 11, 12].

The difference between AC and GDC is not only in interpretation of quantum mechanics. In axiomatic conception of QM all dynamic equations are linear in terms of the world function. This linearity is a conceptual property of the axiomatic conception. In gas dynamical conception of QM dynamic equations are linear in terms of the wave function only for irrotational flow. The axiomatic conception of QM is a conception that does not admit further development. Axiomatic conception supposes that the mean value  $\langle F(\mathbf{x}, \mathbf{p}) \rangle$  of any function  $F(\mathbf{x}, \mathbf{p})$  is determined by the formula

$$\langle F(\mathbf{x}, \mathbf{p}) \rangle = \int \psi^* F(\mathbf{x}, -i\hbar\nabla) \psi d^3\mathbf{x} \quad (4.4)$$

whereas in the gas dynamic conception it is valid only for arbitrary function of coordinates  $F(\mathbf{x})$ , momentum  $\mathbf{p}$ , energy  $E$  and angular momentum  $\mathbf{x} \times \mathbf{p}$ . In other words, GDC supposes, that description in terms of world function is incomplete and it needs a farther development. In nonrelativistic gas dynamics this description of the stochastic molecule motion is specified by introduction of the distribution function  $f(\mathbf{x}, \mathbf{p})$  and kinetic equation for it. The theorem on impossibility of hidden variable in quantum mechanics has been proved by von Neumann at the supposition that relation (4.4) is valid for all functions  $F(\mathbf{x}, \mathbf{p})$ . The gas dynamic conception of quantum mechanics introduces hidden variables, and inside CGC relation (4.4) is valid not for all functions  $F(\mathbf{x}, \mathbf{p})$ .

The GDC of quantum mechanics is specified by introduction of a discrete space-time geometry, where motion of free elementary particles is primordially stochastic [13, 14, 15]. In particular, description of Dirac equation in terms of hydrodynamic variables leads to the particle world line, having a shape of helix [16]. This additional

development leads to so called structural approach to the elementary particles theory [17]. This approach leads to a possibility of investigation of the internal structure of elementary particles, whereas the axiomatic conception of QM leads to so called empirical approach to the elementary particle theory, where the elementary particle is a pointlike object provided by a set of quantum numbers.

It should note, that description in framework of gas dynamic conception admits one to investigate the pair production mechanism. At the reflection a particle from the potential barrier such a  $\kappa$ -field arises near the point of reflection that the coefficient  $K$  (1.4) becomes imaginary. As a result a tachyon region appears near the point of reflection. In this region a pair generation is possible, because in the tachyon region all world lines are spacelike.

Note, that investigation of the  $\kappa$ -field and discrete geometry was produced on the basis of conventional statistical principles [17, 18, 2] without connection with gas dynamics and with kinetic equation. As a result it was impossible to conclude that such a description is more complete, than conventional description in the framework of the axiomatic conception of quantum mechanics. Besides, it was impossible to compare results on the elementary particle structure with results of the standard model of elementary particles, because these results concern to different direction of investigation (different approaches). In the same way one cannot compare the first results on structure of atoms with results of the periodic system of chemical elements, because the atomic structure is determined at the structural approach (atomic physics), whereas the periodic system is determined at the empirical approach (chemistry).

## 5 Summary

It is shown that quantum effects can be described in terms of gas dynamic equations without a use of quantum principles. It means that a reason of quantum effects is some force field, but not quantum principles. As a result the quantum principles are not the prime principles of the nature. It means, that there are fields, which should not be quantized. In particular, there is no necessity to quantize the geometrical fields, for instance, the gravitational field. (In particular, because dynamic equations for the gravitational field do not contain quantum constant). Description of elementary particles in terms of the wave function appears to be incomplete. If one takes into account the space-time geometry discreteness, which is a source of the  $\kappa$ -field, description on the basis of gas dynamic equations may be more complete, than description on the basis of quantum principles,

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