Discriminating properties of compactification in discrete uniform isotropic space-time

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Abstract

Compactification of the 5-dimensional Kaluza-Klein space-time geometry is considered. The space-time geometry is supposed to be discrete, uniform and isotropic. It is shown, that consideration of the space-time geometry as a physical geometry, i.e. as a geometry described completely by the single-valued world function, leads to a discrimination of some values of the particle charge. At the conventional approach, when the world function becomes to be many-valued after compactification, the value of the elementary particle electric charge remains to be unrestricted, and this fact does not agree with experimental data. It is important, that the discrete geometry is given on the continual set of points. This circumstance makes admissible a compatibility of discreteness with the uniformity isotropy of the geometry.

1 Introduction

The role of space-time geometry in description of physical phenomena of microcosm has been increased due to appearance of a more general conception of geometry. In the twentieth century the Riemannian geometry was considered to be the most general geometry, suitable for description of the space-time. However, the Riemannian geometry cannot describe such properties of space-time as discreteness, restricted divisibility of geometrical objects and discrete characteristics (mass, charge, angular momentum) of elementary particles. Discrete characteristics of elementary particles are considered usually to be dynamic properties of elementary particles.

In reality, at a use of a true conception of the space-time geometry the elementary particles in themselves, as well as their properties and their dynamics can be described in terms of the proper space-time geometry and only in terms of the space-time geometry. The conventional conception of geometry, which supposes, that any geometry is axiomatizable, and any geometry can be deduced from a system of axioms, is wrong. In any axiomatizable geometry the equivalence relation is supposed to be transitive. Only at the transitive equivalence relation the set of all geometric propositions (i.e. geometry) can be deduced from an axiomatics (a finite set of basic geometric propositions).

A new method of the physical geometry construction has been invented in the end of the twentieth century [1]. The physical geometry is such a geometry, which is described completely by the world function σ. The world function $\sigma(P,Q)$ is a single-valued real function of any two points $P,Q\in\Omega$, where Ω is the set of all points (or events), where the geometry is given.

$$\sigma: \qquad \Omega\times\Omega\rightarrow\mathbb{R}, \qquad \sigma(P,P)=0, \qquad \forall P\in\Omega \tag{1.1}$$

The world function $\sigma(P,Q)=\frac{1}{2}\rho^2(P,Q)$, where $\rho(P,Q)$ is the distance between the points P and Q.

On one hand, the proper Euclidean geometry $\mathcal{G}_{\rm E}$ is the axiomatizable geometry, which can be deduced from the Euclidean axiomatics [2]. On the other hand, the proper Euclidean geometry $\mathcal{G}_{\rm E}$ is a physical geometry. It means, that all definitions $\mathcal{D}_{\rm E}$ of $\mathcal{G}_{\rm E}$ can be expressed in terms of the Euclidean world function in the form $\mathcal{D}_{\rm E}=\mathcal{D}_{\rm E}[\sigma_{\rm E}]$. There is a theorem, where this statement has been proved [3, 1]. If now one replaces the Euclidean world function $\sigma_{\rm E}$ with the world function σ of some other physical geometry $\mathcal{G}$ in all definitions $\mathcal{D}_{\rm E}:\mathcal{D}_{\rm E}[\sigma_{\rm E}]\rightarrow\mathcal{D}_{\rm E}[\sigma]$, one obtains all definitions $\mathcal{D}_{\rm E}[\sigma]$ of the physical geometry $\mathcal{G}$. The procedure of replacement is a deformation of the proper Euclidean geometry, when the Euclidean distance $\rho_{\rm E}=\sqrt{2\sigma_{\rm E}}$ are replaced by the distance $\rho=\sqrt{2\sigma}$ of the physical geometry $\mathcal{G}$. Thus, any physical geometry is obtained from the proper Euclidean geometry by means of a deformation.

In general, the physical geometry is not axiomatizable, because the axiomatizability of a geometry is possible only, if the equivalence relation is transitive. Indeed, in the proper Euclidean geometry the vector $\mathbf{P}_0\mathbf{P}_1$ is defined as an ordered set $\mathbf{P}_0\mathbf{P}_1=\{P_0,P_1\}$ of two points P_0, P_1. The equivalence (equality) of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by two relations. Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent ($\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1$), if

$$\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1: \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)=|\mathbf{P}_0\mathbf{P}_1|\cdot|\mathbf{Q}_0\mathbf{Q}_1|\wedge|\mathbf{P}_0\mathbf{P}_1|=|\mathbf{Q}_0\mathbf{Q}_1| \tag{1.2}$$

where the scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)=\sigma(P_0,Q_1)+\sigma(P_1,Q_0)-\sigma(P_0,Q_0)-\sigma(P_1,Q_1), \quad \forall P_0,P_1,Q_0,Q_1\in\Omega \tag{1.3}$$

$$|\mathbf{P}_0\mathbf{P}_1|^2=2\sigma(P_0,P_1) \tag{1.4}$$

and σ means the world function of the proper Euclidean geometry. The first relation of (1.2) describes parallelism of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$, whereas the second one describes equality of their lengths. The definition of equivalence of two vectors contains only points P_0, P_1, Q_0, Q_1, determining the vectors, and world functions between these points. The definition does not refer to a coordinate system and to the dimension of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. It is a pure geometric definition, which does not contain a reference to the means of description. In the proper Euclidean geometry the definition (1.2) of equivalence coincides with the conventional equivalence definition on the ground of the linear vector space. The equivalence relation (1.2) is transitive in the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, and this transitivity is a special property of the proper Euclidean geometry.

In the arbitrary physical geometry $\mathcal{G}$ the definition of equivalence has the same form (1.2) with the world function σ, describing the geometry $\mathcal{G}$. However, in the general case the equivalence relation (1.2) is not transitive, in general, because in the case of arbitrary world function σ the equivalence of two vectors is multivariant, in general. It means that at the point P_0 may exist many vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_1'$, $\mathbf{P}_0\mathbf{P}_1''$, ..., which are equivalent to the vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0, whereas the vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_1'$, $\mathbf{P}_0\mathbf{P}_1''$, ...are not equivalent between themselves. In this case it is possible, that

$$\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv} \mathbf{Q}_0\mathbf{Q}_1 \wedge \mathbf{P}_0\mathbf{P}_1' \mathrm{eqv} \mathbf{Q}_0\mathbf{Q}_1 \wedge \mathbf{P}_0\mathbf{P}_1 \overline{\mathrm{eqv}} \mathbf{P}_0\mathbf{P}_1' \tag{1.5}$$

is true. Here the symbol $\overline{\mathrm{eqv}}$ means non-equivalency. If relations (1.5) take place, the equivalence relation is intransitive, because for transitive equivalence relation it follows from

$$\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv} \mathbf{Q}_0\mathbf{Q}_1 \wedge \mathbf{P}_0\mathbf{P}_1' \mathrm{eqv} \mathbf{Q}_0\mathbf{Q}_1 \tag{1.6}$$

that

$$\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv} \mathbf{P}_0\mathbf{P}_1' \tag{1.7}$$

and the relation (1.5) is false. On the other hand, the number of vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_1'$, $\mathbf{P}_0\mathbf{P}_1''$, ..., which are equivalent to the vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0 depends on the number of solutions of two equations (1.2), considered as equations for determination of the point P_1 at fixed points P_0, Q_0, Q_1 (or at fixed point P_0 and fixed vector $\mathbf{Q}_0\mathbf{Q}_1$). The number of these solutions depends on the form of the world function σ. We shall differ three different cases:

(1) Single-variance with respect to points P_0, Q_0, Q_1, when there is one and only one solution for P_1 at given points P_0, Q_0, Q_1. In this case the equivalence relation is transitive, if the single-variance takes place for any points P_0, Q_0, Q_1.

(2) Multivariance with respect to points P_0, Q_0, Q_1, when there is more, than one solution at some given points P_0, Q_0, Q_1. In this case the equivalence relation is intransitive.

(3) zero-variance with respect to points P_0, Q_0, Q_1, when there is no solution at some given points P_0, Q_0, Q_1. In this case the equivalence relation may be intransitive and may be transitive.

The second case is strongest in the sense, that appearance of multivariance with respect to some three points P_0, Q_0, Q_1 generates intransitivity of the equivalence relation, and, hence, non-axiomatizability of a physical geometry, because the transitivity of the equivalence relation is a necessary condition of axiomatizability. The second case and the third one are compatible in the sense that the multivariance may take place with respect to some points P_0, Q_0, Q_1, whereas the zero-variance may take place with respect to other points P_0', Q_0', Q_1'.

Note that the geometry of Minkowski may be axiomatizable and non-physical, and the geometry of Minkowski may be physical and nonaxiomatizable. In general, in this case one has two different geometries, having the same world function. We use for them different names. The geometry of Minkowski, which is a physical geometry, will be referred to as the σ-Minkowskian geometry. The σ-Minkowskian geometry is not axiomatizable, because it is multivariant with respect to any point P_0 and any spacelike vector $\mathbf{Q}_0\mathbf{Q}_1 = \{Q_0, Q_1\}$. The conventional geometry of Minkowski, which is constructed on the ground of the linear vector space with the scalar product, given on it, is axiomatizable (it is deduced from some axiomatics), but it is not a physical geometry. The geometry of Minkowski cannot be constructed on the basis of the world function only. Construction of the geometry of Minkowski contains a reference to the means of description in the form of the coordinate system. Although the construction of the geometry of Minkowski is invariant with respect to transformation of the coordinate system, it is not invariant with respect to transformation of the coordinate system dimension (see detailed discussion in [4]). The geometry of Minkowski should be qualified as a fortified physical geometry, i.e. a physical geometry with some additional structure, given on the physical geometry. Existence of the additional structure imposes some additional constraints on the geometry.

The difference between the space-time geometry of Minkowski and the σ-Minkowskian space-time geometry appears only at consideration of spacelike vectors, with respect to which the σ-Minkowskian geometry is multivariant. However, the spacelike vectors do not figure in the particle dynamics, and the difference between the σ-Minkowskian space-time geometry and the space-time geometry of Minkowski remains to be obscure. If one considers the geometry as a science on mutual disposition of geometrical objects and their shapes, one should prefer the σ-Minkowskian geometry as a space-time geometry, because the distance between any pair of points determines mutual disposition of geometrical objects and their shapes completely. As to axiomatizability of a geometry, this property is important only for deduction of the geometric propositions from the axiomatics. From viewpoint of the geometry as a science on disposition of geometrical object, the axiomatizability is a secondary property of the geometry, and practically all physical geometries are not axiomatizable. The proper Euclidean geometry is a very important exclusion, which admits one to construct physical geometries by means of a deformation of the proper Euclidean geometry.

Deduction of an axiomatizable geometry from axiomatics has two essential defects. Firstly, one needs to formulate geometric propositions and to prove corresponding theorems. The geometric propositions are to be formulated and proved for any new geometry. These procedures are complicated from the technical viewpoint. Besides, only geometries with the transitive equivalence relation can be deduced from axiomatics. Secondly, one needs to invent axioms, and to test their consistency. Inconsistency of a geometry means, that using two different ways of deduction of some statement, one obtains two incompatible statements. *Inconsistency of a geometry is a property of the method of the geometry construction, but not a property of the geometry in itself.* In the physical geometry, which is constructed on the ground of the deformation principle, the question of its inconsistency is meaningless, because the problem of geometric propositions formulation is absent at all. All definitions of a physical geometry are taken from the proper Euclidean geometry in the ready-made form. More exactly, definitions of geometrical objects are taken from the proper Euclidean geometry. If it is necessary to determine properties of these objects, they are calculated on the basis of the world function. As far as the world functions are different, in general, in the considered geometry and in the Euclidean one, the calculated properties may be different in the considered geometry and in the Euclidean one.

Finally, the method of a physical geometry construction, based on linear vector space, (for instance, construction of the Riemannian geometry) starts from some n-dimensional manifold $\mathcal{M}_n$, where the metric tensor g_{ik} is given. The world function σ is given by the relation

$$\sigma\left(x,x'\right)=\frac{1}{2}\left(\int\limits_{\mathcal{L}_{xx'}}\sqrt{g_{ik}\left(x\right)dx^{i}dx^{k}}\right)^{2} \tag{1.8}$$

where integration is produced along the geodesic $\mathcal{L}_{xx'}$, connecting points x and x'. There may be several geodesics, connecting points x and x'. In this case the world function σ appears to be many-valued. In this case the world function is a derivative quantity, and it may be many-valued. However, in a physical geometry, the world function is a primary quantity, it determines the physical geometry, and it cannot be many-valued.

To make the Riemannian geometry with many-valued world function (1.8) a physical geometry (σ-Riemannian geometry), one needs to turn the many-valued world function into single-valued one, choosing only one branch of the world function (1.8). Different choice of branches generates different world functions and, hence, different σ-Riemannian geometries. Thus, the n-dimensional manifold $\mathcal{M}_n$ with the metric tensor, given on it generates several σ-Riemannian geometries, if the expression (1.8) appears to be many-valued for some pairs of points x, x'.

Construction of σ-Riemannian geometries by means of a transformation of many-valued world function (1.8) into a single-valued world function is accompanied by appearance of zero-variance for some points. Of course, this mechanism of construction of a physical geometry with the zero-variance is not unique. However, this mechanism is interesting from the physical viewpoint, because the σ-Riemannian geometry with the zero-variance may be obtained as a result of the compactification of the flat space-time geometry (for instance, compactification of 5-dimensional space-time geometry of Kaluza-Klein [6, 7]). The zero-variance generates some discrimination mechanism, responsible for discrete values of the elementary particle parameters. In particular, compactification of the fifth coordinate in the Kaluza-Klein geometry leads to restrictions on the possible electric charge of the elementary particle.

This paper is devoted to consideration of the procedure of compactification of the Kaluza-Klein geometry, which is accompanied by the construction of a discrimination mechanism, imposing restrictions on the value of the electric charge of the elementary particles. However, at first, we mention about influence of the multivariance upon the dynamics of elementary particles.

2 Influence of the multivariance on the particle dynamics.

In the space-time geometry of Minkowski the dynamics of a pointlike particle is described by a timelike world line $\mathcal{L}$ of the particle. In the inertial coordinate system $x = \{x^0, x^1, x^2, x^3\}$ the world function $\sigma_{\mathrm{M}}(x, x')$ between two points with coordinates x and x' has the form

$$\sigma_{\mathrm{M}}(x, x') = \frac{1}{2} g_{ik} \left(x^i - x'^i\right) \left(x^k - x'^k\right) \tag{2.1}$$

where the metric tensor has the form $g_{ik} = \mathrm{diag}\{c^2, -1, -1, -1\}$, and c is the speed of the light. The world line $x = x(\tau)$ of a charged particle, moving in the given electromagnetic field F_{ik}, is described by the dynamic equation

$$m \frac{d}{d\tau} \frac{c g_{il} \frac{dx^l}{d\tau}}{\sqrt{g_{jn} \frac{dx^j}{d\tau} \frac{dx^n}{d\tau}}} = \frac{e}{c} F_{ik}(x) \frac{dx^k}{d\tau}, \qquad i = 0, 1, 2, 3 \tag{2.2}$$

where m is the particle mass, e is the electric charge of the particle and τ is a parameter along the world line. The constants m and e are non-geometrical characteristics of the pointlike particle.

In general, the mass m and the charge e can be geometrized, i.e. they may be considered as pure geometric characteristics of the pointlike particle. However, it is possible only in the framework of the physical geometry, which is formulated in terms of the world function. The motion of a pointlike particle is described by a world chain $\mathcal{C}$, consisting of connected vectors $\mathbf{P}_s\mathbf{P}_{s+1}$, $s = ...0, 1, ...$

$$\mathcal{C} = \bigcup_s \mathbf{P}_s\mathbf{P}_{s+1} \tag{2.3}$$

where vector $\mathbf{P}_s\mathbf{P}_{s+1} = \{P_s, P_{s+1}\}$ is an ordered set of two points P_s, P_{s+1}. The world chain $\mathcal{C}$ is an ordered set of points $...P_0, P_1, ...P_s, ...$ The distance $|\mathbf{P}_s\mathbf{P}_{s+1}|$ between the adjacent points P_s, P_{s+1} is the same.

$$|\mathbf{P}_s\mathbf{P}_{s+1}| = \mu, \qquad s = ...0, 1, .. \tag{2.4}$$

The quantity $\mu = |\mathbf{P}_s\mathbf{P}_{s+1}|$ is the length of the world chain link. It determines the geometric mass μ. The geometrical mass μ is connected with the usual mass m by means of the relation

$$m = b\mu \tag{2.5}$$

where b is some universal constant. The geometrical mass μ is a geometric characteristic of the particle, as well as the vector $\mathbf{P}_s\mathbf{P}_{s+1}$, which is the geometric momentum of the particle.

The motion (2.2) of a pointlike particle in the electromagnetic field may be described as a free motion of the particle in the 5-dimensional space-time of Kaluza-Klein. The fact, that the motion of a pointlike particle in a physical space-time geometry is free, means that the adjacent vectors in the world chain are equivalent

$$\mathbf{P}_s\mathbf{P}_{s+1}\text{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}, \qquad s = ...0, 1, .. \tag{2.6}$$

Let the electromagnetic field be absent. Then dynamic equation (2.2) turns into the dynamic equation

$$m\frac{d}{d\tau}\frac{cg_{il}\frac{dx^l}{d\tau}}{\sqrt{g_{jn}\frac{dx^j}{d\tau}\frac{dx^n}{d\tau}}} = 0 \tag{2.7}$$

Its solution

$$x^i = x^i(\tau) = X^i + U^i\tau, \qquad X^i, U^i = \text{const}, \qquad i = 0, 1, 2, 3 \tag{2.8}$$

does not depend on the mass m and coincides with the solution of equations (2.6), (1.2) in the space-time of Minkowski. However, if the space-time of Minkowski is slightly deformed, the solution may appear to depend on the mass.

Let us consider the space-time geometry $\mathcal{G}_{\rm d}$, described by the world function

$$\sigma_{\rm d} = \sigma_{\rm M} + d, \qquad d = \frac{1}{2}\lambda_0^2\text{sgn}(\sigma_{\rm M}), \qquad \lambda_0 = \text{const} \tag{2.9}$$

$$\text{sgn}(x) = \begin{cases} \frac{x}{|x|} & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases} \tag{2.10}$$

where $\sigma_{\rm M}$ is the world function of the space-time geometry of Minkowski, and λ_0 is some elementary length of the geometry $\mathcal{G}_{\rm d}$.

The length $|\mathbf{P}_0\mathbf{P}_1|_{\rm d}$ of any vector $\mathbf{P}_0\mathbf{P}_1$ has the form

$$|\mathbf{P}_0\mathbf{P}_1|_{\rm d}^2 = 2\sigma_{\rm d}(P_0, P_1) = 2\sigma_{\rm M}(P_0, P_1) + \lambda_0^2\text{sgn}(\sigma_{\rm M}(P_0.P_1)) \tag{2.11}$$

In other words, if the distance between points P_0, P_1 is timelike in $\mathcal{G}_{\rm d}$ $(\sigma_{\rm d}(P_0, P_1) > 0)$, it is also timelike in $\mathcal{G}_{\rm M}$ $(\sigma_{\rm M}(P_0, P_1) > 0)$. If the distance between points P_0, P_1 is spacelike in $\mathcal{G}_{\rm d}$ $(\sigma_{\rm d}(P_0, P_1) < 0)$, it is also spacelike in $\mathcal{G}_{\rm M}$ $(\sigma_{\rm M}(P_0, P_1) < 0)$. It follows from (2.11) that any timelike (and spacelike) distance is larger, than λ_0. It means that in the space-time geometry $\mathcal{G}_{\rm d}$ there are no close points, and the geometry $\mathcal{G}_{\rm d}$ should be qualified as a discrete space-time geometry. The geometry $\mathcal{G}_{\rm d}$ is given on the continuous manifold of Minkowski. It looks rather unexpected, that the discrete geometry may be given on the same point set, on which a continuous geometry can be given. This surprise is explained by the fact, that at the conventional approach, based on the concept of the linear space, the discrete geometry is given on a countable point set, whereas the continuous geometry is given on a continual point set.

Conventionally a discrete geometry is described as follows. Let us consider some geometry $\mathcal{G}_{\rm c}$ (Euclidean, Minkowskian, or Riemannian) on some manifold $\mathbb{M}_n$ and introduce some curvilinear coordinate system $(x^0, x^1, ...x^n)$ in it. Let us remove from the manifold $\mathbb{M}_{n+1}$ all points except of points with all integer coordinates. As a result one obtains the point set $\mathbb{M}_{\rm d}$, whose points a labelled by integer coordinates x^s, $s = 0, 1, , ..n$. The world function $\sigma(P, Q)$ between the points $P, Q \in \mathbb{M}_{\rm d}$ is the same as between the corresponding points $P, Q \in \mathbb{M}_{n+1}$. As a result one obtains the same geometry $\mathcal{G}_{\rm c}$ on the subset $\mathbb{M}_{\rm d}$ of the set $\mathbb{M}_{n+1}$. In the discrete geometry $\mathcal{G}_{\rm c}$ defined on $\mathbb{M}_{\rm d}$ there is an elementary length λ, defined by the relation

$$\lambda = \min_{\forall P,Q\in\mathbb{M}_{\rm d}} \left\{ \left| \sqrt{2\sigma (P,Q)} \right| \right\} \quad \text{at} \quad \left| \sqrt{2\sigma (P,Q)} \right| > 0 \tag{2.12}$$

In this conventional definition of the discrete geometry one uses such means of description as the manifold $\mathbb{M}_{n+1}$ and a coordinate system on it. The obtained geometry on $\mathbb{M}_{\rm d}$ depends essentially on the choice of the coordinate system. Besides, it is impossible to obtain a discrete geometry on a continuous set of points.

The definition (2.9) does not use any means of description. It uses only world function, and discreteness of the geometry arises from the fact that $|\sigma (P, Q)| \notin (0, \lambda_0)$ for $\forall P, Q \in \Omega$, where Ω is the point set where the geometry is given. The set Ω may be discrete or continuous. This circumstance is unessential for construction of the discrete geometry.

The character (discreteness, or continuity) of geometry depends only on the form of the world function. Of course, the continual geometry may be given only on a continual point set. However, as we have seen, the discrete geometry may be given also on a continual point set .

As we have seen, the σ-Minkowskian geometry is multivariant with respect to any point and any spacelike vector. The space-time geometry $\mathcal{G}_{\rm d}$ is multivariant with respect to timelike vectors also, and this circumstance appears to be important for dynamics of a pointlike particle, because the dynamics deals with timelike vectors. The free motion of a pointlike particle appears to depend on the geometric particle mass μ and on the elementary length λ_0, which is responsible for multivariance of $\mathcal{G}_{\rm d}$ with respect to timelike vectors.

Two adjacent links $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ are equivalent, and, hence, satisfy the relations of the type of (1.2). Let coordinates of the points be

$$P_0 = \{0, 0, 0, 0\}, \qquad P_1 = \{\mu, 0, 0, 0\}, \qquad P_2 = \{2\mu + \alpha_0, \alpha_1, \alpha_2, \alpha_3\} \tag{2.13}$$

The coordinates of vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_0\mathbf{P}_2$ are

$$\mathbf{P}_0\mathbf{P}_1 = \{\mu, 0, 0, 0\}, \qquad \mathbf{P}_1\mathbf{P}_2 = \{\mu + \alpha_0, \alpha_1, \alpha_2, \alpha_3\}, \tag{2.14}$$

$$\mathbf{P}_0\mathbf{P}_2 = \{2\mu + \alpha_0, \alpha_1, \alpha_2, \alpha_3\} \tag{2.15}$$

Let us take into account that

$$|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{d}}^2 = |\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2 = 2\sigma_{\mathrm{M}}(P_0, P_1) + \lambda_0^2 \mathrm{sgn}(\sigma_{\mathrm{M}}(P_0, P_1)) \tag{2.16}$$

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\mathrm{d}} = (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\mathrm{M}} + w(P_0, P_1, P_1, P_2) \tag{2.17}$$

Here indices "M" and "d" mean that the quantities are calculated in $\mathcal{G}_{\mathrm{M}}$ and $\mathcal{G}_{\mathrm{d}}$ respectively, and for timelike vectors (2.14)

$$w(P_0, P_1, P_1, P_2) = d(P_0, P_2) + d(P_1, P_1) - d(P_0, P_1) - d(P_1, P_2) = -\frac{1}{2}\lambda_0^2 \tag{2.18}$$

The relation $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{P}_1\mathbf{P}_2$ has the form of two equations

$$\mu(\mu + \alpha_0) - \frac{1}{2}\lambda_0^2 = \mu^2 + \lambda_0^2 \tag{2.19}$$

$$\mu^2 = (\mu + \alpha_0)^2 - \alpha_1^2 - \alpha_2^2 - \alpha_3^2 \tag{2.20}$$

The quantities α are to be determined from these equations. Solution of equations (2.19), (2.20) has the form

$$\alpha_0 = \frac{3\lambda_0^2}{2\mu}, \qquad \alpha_1 = \lambda_0\sqrt{3 + \frac{9\lambda_0^2}{4\mu^2}} \sin\theta\cos\varphi, \tag{2.21}$$

$$\alpha_2 = \lambda_0\sqrt{3 + \frac{9\lambda_0^2}{4\mu^2}} \sin\theta\sin\varphi, \qquad \alpha_3 = \lambda_0\sqrt{3 + \frac{9\lambda_0^2}{4\mu^2}} \cos\theta \tag{2.22}$$

where the quantities θ and φ are arbitrary.

Thus, position of the link $\mathbf{P}_1\mathbf{P}_2$ with respect to the adjacent link $\mathbf{P}_0\mathbf{P}_1$ appears to be indefinite (multivariant). Possible positions of the link $\mathbf{P}_1\mathbf{P}_2$ form generatrices of the cone with the axis $\mathbf{P}_0\mathbf{P}_1$and the angle ϕ at the vertex, which lies at the point P_1. The angle ϕ is determined by the relation

$$\tan\phi = \frac{\sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}}{\mu + \alpha_0} = \frac{\lambda_0}{\mu\left(1 + \frac{3\lambda_0^2}{\mu^2}\right)}\sqrt{3 + \frac{9\lambda_0^2}{4\mu^2}} \approx \frac{\lambda_0\sqrt{3}}{\mu}, \qquad \text{if } \lambda_0 \ll \mu \tag{2.23}$$

If the elementary length $\lambda_0 \to 0$, the space-time geometry $\mathcal{G}_{\mathrm{d}}$ turns into $\mathcal{G}_{\mathrm{M}}$, and the cone degenerates into a straight line.

Indefinite (multivariant) position of adjacent links leads to wobbling of the world chain of the pointlike particle. Let us choose elementary length of the space-time geometry $\mathcal{G}_{\mathrm{d}}$ in the form

$$\lambda_0^2 = \frac{\hbar}{bc} \tag{2.24}$$

where $\hbar$ is the quantum constant, c is the speed of the light and the constant b is the universal constant (2.5), connecting the geometrical mass μ with the usual mass m. Then the statistical description of wobbling world chains is equivalent to the quantum description in terms of the Schrödinger equation [5]. The quantum constant $\hbar$ appears in the dynamics of the particle as a parameter of the space-time geometry $\mathcal{G}_{\rm d}$. The conventional quantum principles appear to be needless. Thus, the multivariant space-time geometry admits one to describe quantum effects as geometric effects. Besides, the pointlike particle mass m appears to be geometrized by its connection (2.5) with the geometrical mass $\mu = |\mathbf{P}_0\mathbf{P}_1|_{\rm d}$.

We have no direct information on the space-time geometry in microcosm. In the usual scale the space-time geometry may be considered as continuous, because the possible discreteness of the space-time has a small scale, which cannot be recognized in macroscopic experiments. However, in the small scale the space-time geometry may appear to be discrete. The discrete space-time geometry generates multivariance, which is responsible for quantum effects. It is impossible to object anything against the discreteness of the space-time geometry at small scale. Such a possibility should be considered. The discrete space-time geometry is to be considered in the framework of the physical geometry, which describes continuous and discrete geometries, using a uniform method.

3 World function of the Kaluza-Klein space-time

The space-time geometry of Kaluza-Klein $\mathcal{G}_{\rm K}$ is given on the 5-dimensional manifold. In the coordinate system with coordinates $x = \{x^0, x^1, x^2, x^3, x^5\}$. Four coordinates $\{x^0, x^1, x^2, x^3\} = \{x^0, \mathbf{x}\}$ describe position of a particle in the 4D-space-time of Minkowski, whereas the charge coordinate x^5 describes additional characteristic of the particle, which is responsible for interaction with the electromagnetic field.

Covariant metric tensor γ_{AB}, $A, B = 0, 1, 2, 3, 5$ in the geometry $\mathcal{G}_{\rm K}$ is determined by the relation

$$\gamma_{AB} = \left\| \begin{array}{cc} g_{ik} - a_i a_k & a_k \\ a_i & -1 \end{array} \right\|, \qquad i, k = 0, 1, 2, 3, \qquad A, B = 0, 1, 2, 3, 5 \qquad (3.1)$$

where g_{ik}, $i, k = 0, 1, 2, 3$ is the metric tensor in the conventional 4-dimensional space-time. The quantities a_k, $k = 0, 1, 2, 3$ are connected with electromagnetic potential A_k, $k = 0, 1, 2, 3$ by means of the relation

$$a_k = \varkappa A_k, \qquad k = 0, 1, 2, 3 \qquad (3.2)$$

where $\varkappa$ is some universal constant. The contravariant metric tensor γ^{AB}, $A, B = 0, 1, 2, 3, 5$ has the form

$$\gamma^{AB} = \left\| \begin{array}{cc} g^{ik} & g^{il} a_l \\ g^{kl} a_l & -1 + g^{jl} a_j a_l \end{array} \right\|, \qquad i, k = 0, 1, 2, 3, \qquad A, B = 0, 1, 2, 3, 5 \quad (3.3)$$

It is supposed that neither electromagnetic potentials a_k, nor the metric tensor g_{ik} depend on the charge coordinate x^5.

Then the action

$$\mathcal{A}[x] = \int \left\{ -m_5 c \sqrt{\gamma_{AB} \dot{x}^A \dot{x}^B} \right\} d\tau, \qquad x = \left\{ x^0(\tau), x^1(\tau), x^2(\tau), x^3(\tau), x^5(\tau) \right\} \tag{3.4}$$

describes the motion of a charged particle in the gravitational field,described by the metric tensor g_{ik} and in the electromagnetic field A_k. Corresponding dynamic equations are obtained as a result of variation of the action (3.4) with respect to x^A, $A = 0, 1, 2, 3, 5$.

$$\frac{dp_A}{d\tau} = -\frac{\partial}{\partial x^A} \left(m_5 c \sqrt{\gamma_{AB} \dot{x}^A \dot{x}^B} \right), \qquad A = 0, 1, 2, 3, 5 \tag{3.5}$$

where

$$p_A = -\frac{m_5 c \gamma_{AB} \dot{x}^B}{\sqrt{\gamma_{CD} \dot{x}^C \dot{x}^D}}, \qquad A = 0, 1, 2, 3, 5 \tag{3.6}$$

As far as γ_{AB} does depend on x^5, it follows from (3.5), that the canonical momentum component p_5 =const. Then, taking into account (3.1), the equation (**??**) may be rewritten in the form

$$\left(\frac{\partial S}{\partial x^i} + p_5 a_i \right) g^{ik} \left(\frac{\partial S}{\partial x^k} + p_5 a_k \right) = (m_5 c)^2 + p_5^2 \tag{3.7}$$

Comparing (3.7) with the Hamilton-Jacobi equation

$$\left(\frac{\partial S}{\partial x^i} + \frac{e}{c} A_i \right) g^{ik} \left(\frac{\partial S}{\partial x^k} + \frac{e}{c} A_k \right) = m^2 c^2 \tag{3.8}$$

describing motion of a pointlike particle of mass m and of charge e in 4-dimensional space-time with electromagnetic potential A_k, $k = 0, 1, 2, 3$, one concludes that equations (3.7) and (3.8) are equivalent, if

$$m = \sqrt{m_5^2 + c^{-2} p_5^2}, \qquad p_5 = \frac{e}{\varkappa c}, \qquad a_k = \varkappa A_k, \qquad k = 0, 1, 2, 3 \tag{3.9}$$

where $\varkappa$ is some universal constant.

The original action (3.4) has the form of the action for a geodesic in 5-dimensional Riemannian space with the metric tensor (3.1). Thus, the motion of a pointlike charged particle in the 4-dimensional Riemannian space-time with the electromagnetic field can be described as a free motion of a particle in the 5-dimensional Riemannian space-time. The electric charge e of the particle is geometrized in the sense, that it appears to be connected with the component p_5 of the particle momentum along the fifth (charge) coordinate x^5.

However, the fifth coordinate x^5 is unobservable, and one tries to explain this circumstance by the hypothesis, that the space-time of Kaluza-Klein is thin in the direction of the fifth coordinate x^5. One supposes, that the space-time of Kaluza-Klein is compactified in the direction of fifth coordinate x^5, i.e. the points with coordinates $\{x^0, x^1, x^2, x^3, x^5\}$ and $\{x^0, x^1, x^2, x^3, x^5 + 2kL\}$ coincide, where L is some universal constant and k is any integer number.

4 Discrimination properties of the Kaluza-Klein geometry compactification

We shall try to analyze influence of compactification on the Kaluza-Klein geometry $\mathcal{G}_{\mathrm{K}}$. For simplicity we shall consider the case, when the gravitational field and the electromagnetic one are absent. Then the metric tensor (3.1) takes the form γ_{AB} =diag$(c^2,-1,-1,-1,-1)$ and $a_k=0$, $k=0,1,2,3$. Geodesics $\mathcal{L}_{P_0P_1}$, passing through points P_0 and P_1 with coordinates

$$P_0=\{0,0,0,0,0\}\,,\qquad P_1=\left\{y^0,y^1,y^2,y^3,y^5\right\},\qquad y^0,y^1,y^2,y^3,y^5\in\mathbb{R} \tag{4.1}$$

have the form

$$x^k=y^k\tau,\qquad x^5=\left(y^5+2nL\right)\tau,\qquad k=0,1,2,3 \tag{4.2}$$

where τ is a parameter along the geodesic, and n is an arbitrary integer number. The compactification may be considered as a conglutination of points with coordinates $\{x^0,x^1,x^2,x^3,x^5-L\}$ and $\{x^0,x^1,x^2,x^3,x^5+L\}$. As a result one obtains a "cylinder" instead of a plane. The compactification distinguishes the space-time direction of the coordinate x^5 in the sense that it forbids space-time rotations, including the coordinate x^5.

Defining the world function $\sigma_{\mathrm{K}}(P_0,P_1)$ by means of (1.8) as an integral along the geodesic, connecting points P_0 and P_1, one obtains a many-valued world function, because there are many geodesics of different length, connecting the points P_0 and P_1. If the space-time geometry is constructed according to conventional method on the basis of the linear vector space, the metric tensor is a primary quantity, whereas the world function is a secondary (derivative) quantity. In this case one may accept situation with many-valued world function, and one may try to interpret this fact in some way.

However, if the space-time geometry is a physical geometry, where the world function is the primary fundamental quantity, one cannot accept a many-valued primary quantity. One needs to use a single-valued world function and to choose only one of many possible variants of the geodesic (4.2). One obtains different space-time geometries for different choice of the geodesic (4.2), determining the world function.

The single-valued world function restricts possible values of electric charge, considered as a momentum along the fifth coordinate x^5 in the space-time of Kaluza-Klein. As a result of the single-valued world function the electric charge of an elementary particle appears to be restricted. Compactification with many-valued world function does not need such a restriction.

We consider the simplest case, when the world function is defined as integral (1.8) along the "shortest" geodesic, corresponding to the geodesic (4.2). This geodesic makes less, than one convolution around the "cylinder". In this case the world function depends on the standartized value x^5_{st} of the coordinate x^5

$$\sigma_{\mathrm{K}}(x,x')=\frac{1}{2}\left(\left(x^0-x'^0\right)^2-\left(\mathbf{x}-\mathbf{x}'\right)^2-\left(\left(x^5-x'^5\right)_{\mathrm{st}}\right)^2\right) \tag{4.3}$$

where $\mathbf{x} = \{x^1, x^2, x^3\}$

$$x_{\text{st}} = \left\{ \begin{array}{lll} 2L\left\{\frac{x}{2L}\right\} & \text{if} & 2L\left\{\frac{x}{2L}\right\} \leq L \\ 2L\left\{\frac{x}{2L}\right\} - 2L & \text{if} & L < 2L\left\{\frac{x}{2L}\right\} \end{array} \right., \qquad 2L\left\{\frac{x}{2L}\right\} \in [0, 2L) \tag{4.4}$$

Here $\{x\}$ means the fractional part of a decimal number x, and $[x]$ is the integer part of x. In other words, $[x]$ and $\{x\}$ are defined by relations.

$$[x] = \max\left(k \in \mathbb{Z} | k \leq x\right), \tag{4.5}$$

where $\mathbb{Z}$ is the set of all integer numbers.

$$\{x\} = x - [x] \tag{4.6}$$

The coordinate x_{st}^5 is a standartized coordinate $x_{\text{st}}^5 \in (-L, L]$, although formally $x^5 \in \mathbb{R}$. The expression $(x^5 - x'^5)_{\text{st}} \in (-L, L]$, although formally $x^5, x'^5 \in \mathbb{R}$. We have

$$(x_{\text{st}} - x'_{\text{st}})_{\text{st}} = \left\{ \begin{array}{lll} x_{\text{st}} - x'_{\text{st}} & \text{if} & -L < x_{\text{st}} - x'_{\text{st}} \leq L \\ -2L + x_{\text{st}} - x'_{\text{st}} & \text{if} & L < x_{\text{st}} - x'_{\text{st}} \leq 3L \\ 2L + x_{\text{st}} - x'_{\text{st}} & \text{if} & -3L < x_{\text{st}} - x'_{\text{st}} \leq -L \end{array} \right. \tag{4.7}$$

The choice of the world function σ_{K} in the form (4.3), (4.4) corresponds to the geodesic (4.2), which makes less, than one convolution around the "cylinder". The world function (4.3), (4.4) is zero-variant with respect to some vectors.

Let us consider the points of two adjacent vectors of a world chain.

$$P_0 = \{0,0,0,0,0\}, \qquad P_1 = \{s_0, s_1, s_2, s_3, l\}, \tag{4.8}$$

$$P_2 = \{2s_0 + \alpha_0, 2s_1 + \alpha_1, 2s_2 + \alpha_2, 2s_3 + \alpha_3, 2l + \alpha_5\} \tag{4.9}$$

$$\mathbf{P}_0\mathbf{P}_1 = s = \{s_0, s_1, s_2, s_3, l\}, \tag{4.10}$$

$$\mathbf{P}_1\mathbf{P}_2 = s + \alpha = \{s_0 + \alpha_0, s_1 + \alpha_1, s_2 + \alpha_2, s_3 + \alpha_3, l + \alpha_5\} \tag{4.11}$$

$$\mathbf{P}_0\mathbf{P}_2 = 2s + \alpha = \{2s_0 + \alpha_0, 2s_1 + \alpha_1, 2s_2 + \alpha_2, 2s_3 + \alpha_3, 2l + \alpha_5\} \tag{4.12}$$

We shall show, that if the fifth coordinate $x^5 = l$ satisfies the relation

$$|l| > \frac{L}{2} \tag{4.13}$$

then the vector $\mathbf{P}_1\mathbf{P}_2$, which is equivalent to vector $\mathbf{P}_0\mathbf{P}_1$ does not exist. It means, that the world chain of a free pointlike particle with the link $\mathbf{P}_0\mathbf{P}_1$ cannot exist.

The equivalence conditions $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{P}_1\mathbf{P}_2$ for vectors (4.10), (4.11) are written in the form

$$|\mathbf{P}_0\mathbf{P}_1|_{\text{K}}^2 = |\mathbf{P}_1\mathbf{P}_2|_{\text{K}}^2 \tag{4.14}$$

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\text{K}} = |\mathbf{P}_0\mathbf{P}_1|_{\text{K}}^2 \tag{4.15}$$

where index "K" means, that the corresponding quantities are taken in the geometry (4.3).

We suppose, that the vector $\mathbf{P}_0\mathbf{P}_1$ is timelike in the sense, that

$$s_0^2 > L^2 + \mathbf{s}^2, \qquad \mathbf{s} = \{s_1, s_2, s_3\} \tag{4.16}$$

As far as

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\mathrm{K}} = \sigma_{\mathrm{K}}(P_0, P_2) - \sigma_{\mathrm{K}}(P_0, P_1) - \sigma_{\mathrm{K}}(P_1, P_2) \tag{4.17}$$

the equations (4.14), (4.15) are written in the form

$$s_0^2 - \mathbf{s}^2 - l^2 = (s_0 + \alpha_0)^2 - (\mathbf{s} + \boldsymbol{\alpha})^2 - ((l + \alpha_5)_{\mathrm{st}})^2 \tag{4.18}$$

$$(2s_0 + \alpha_0)^2 - (2\mathbf{s} + \boldsymbol{\alpha})^2 - (2l + \alpha_5)_{\mathrm{st}}^2 = 4\left(s_0^2 - \mathbf{s}^2 - l^2\right) \tag{4.19}$$

Taking sum of equations (4.19) and (4.18), one obtains

$$2s_0\alpha_0 - 2\mathbf{s}\boldsymbol{\alpha} - (2l + \alpha_5)_{\mathrm{st}}^2 + (l + \alpha_5)_{\mathrm{st}}^2 = -3l^2 \tag{4.20}$$

$$\alpha_0 = \frac{2\mathbf{s}\boldsymbol{\alpha} + (2l + \alpha_5)_{\mathrm{st}}^2 - (l + \alpha_5)_{\mathrm{st}}^2 - 3l^2}{2s_0} \tag{4.21}$$

Substituting (4.21) in (4.18), one obtains

$$\boldsymbol{\alpha}^2 = (2l + \alpha_5)_{\mathrm{st}}^2 - 2(l + \alpha_5)_{\mathrm{st}}^2 - 2l^2 + \left(\frac{2\mathbf{s}\boldsymbol{\alpha} + (2l + \alpha_5)_{\mathrm{st}}^2 - (l + \alpha_5)_{\mathrm{st}}^2 - 3l^2}{2s_0}\right)^2 \tag{4.22}$$

Let us set

$$\beta = \beta_{\mathrm{st}} = (l + \alpha_5)_{\mathrm{st}} \tag{4.23}$$

Then

$$(2l + \alpha_5)_{\mathrm{st}} = (l + \beta)_{\mathrm{st}} = l + \beta + \gamma \tag{4.24}$$

where

$$\gamma = \left\{\begin{array}{ccc} 0 & \text{if} & -L < l + \beta \leq L \\ -2L & \text{if} & L < l + \beta \leq 3L \\ 2L & \text{if} & -3L < l + \beta \leq -L \end{array}\right. = \left\{\begin{array}{ccc} 0 & \text{if} & -L - l < \beta \leq L - l \\ -2L & \text{if} & L - l < \beta \leq 3L - l \\ 2L & \text{if} & -3L - l < \beta \leq -L - l \end{array}\right. \tag{4.25}$$

Note, that we are interested in the quantity β, because it is the fifth coordinate of the vector $\mathbf{P}_1\mathbf{P}_2$, which is determined to within $2kL$, where k is an arbitrary integer number

$$\begin{aligned} \mathbf{P}_1\mathbf{P}_2 &= \{s_0 + \alpha_0, s_1 + \alpha_1, s_2 + \alpha_2, s_3 + \alpha_3, \beta\} \\ &= \{s_0 + \alpha_0, s_1 + \alpha_1, s_2 + \alpha_2, s_3 + \alpha_3, \beta + 2kL\} \end{aligned} \tag{4.26}$$

Substituting (4.23) and (4.24) in (4.22), one obtains after transformations

$$\begin{aligned} \boldsymbol{\alpha}^2 \;=\; & +\frac{2\mathbf{s}\boldsymbol{\alpha} l(\beta - l)}{s_0^2} + \frac{(\mathbf{s}\boldsymbol{\alpha})^2}{s_0^2} - (l - \beta)^2\left(1 - \frac{l^2}{s_0^2}\right) + \gamma^2\left(\frac{\frac{1}{2}\gamma + (l + \beta)}{s_0}\right)^2 \\ & + \gamma(\gamma + 2(l + \beta))\left(1 + \frac{l(\beta - l) + \mathbf{s}\boldsymbol{\alpha}}{s_0^2}\right) \end{aligned} \tag{4.27}$$

Let us consider the case, when

$$\gamma = 0, \qquad -L < \beta + l \leq L, \tag{4.28}$$

Then one obtains from (4.27)

$$\boldsymbol{\alpha}^2 = -(l-\beta)^2 \left(1 - \frac{l^2}{s_0^2}\right) + \left(\frac{\mathbf{s}\boldsymbol{\alpha}}{s_0}\right)^2 - 2\frac{\mathbf{s}\boldsymbol{\alpha}}{s_0^2} l\,(l-\beta) \tag{4.29}$$

One can see, that the equation (4.29) has the evident solution

$$\beta = l, \qquad \boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3) = (0,0,0)\,, \qquad \alpha_0 = 0 \tag{4.30}$$

. It follows from (4.28) and (4.30), that

$$-L/2 < l \leq L/2, \qquad -L/2 < \beta \leq L/2 \tag{4.31}$$

To obtain other solutions, let us set

$$\beta = l + \varepsilon \tag{4.32}$$

One obtains instead of (4.29)

$$\boldsymbol{\alpha}^2 = -\varepsilon^2 \left(1 - \frac{l^2}{s_0^2}\right) + \left(\frac{\mathbf{s}\boldsymbol{\alpha}}{s_0}\right)^2 + 2\frac{\mathbf{s}\boldsymbol{\alpha}}{s_0^2} l\varepsilon \tag{4.33}$$

Or

$$\sum_\beta \left(s_0^2\alpha_\beta^2 - 2ls_\beta\varepsilon\alpha_\beta - \sum_\nu s_\beta s_\nu \alpha_\beta \alpha_\nu\right) + \varepsilon^2\left(s_0^2 - l^2\right) = 0 \tag{4.34}$$

We are to find such spacelike vectors $\{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \varepsilon\}$ and such a value of the variable l, which satisfy the equation (4.34).

Let us choose the axis x^1 along the 3-vector $\mathbf{s}$. Equation (4.34) takes the form

$$\left(s_0^2 - s_1^2\right)\alpha_1^2 + s_0^2\alpha_2^2 + s_0^2\alpha_3^2 + \left(s_0^2 - l^2\right)\varepsilon^2 - 2ls_1\varepsilon\alpha_1 = 0 \tag{4.35}$$

Lhs of equation (4.35) is a quadratic form with respect to variables $\{\alpha_1, \alpha_2, \alpha_3, \varepsilon\}$ .The matrix of the quadratic form of the equation (4.35) has the form

$$\left\|\begin{array}{cccc} s_0^2 - s_1^2 & 0 & 0 & -ls_1 \\ 0 & s_0^2 & 0 & 0 \\ 0 & 0 & s_0^2 & 0 \\ -ls_1 & 0 & 0 & (s_0^2 - l^2) \end{array}\right\| \tag{4.36}$$

Eigenvectors and eigenvalues of the quadratic form (4.36) have the form,

$$\left\{\begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \end{array}, \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array}, \begin{array}{c} -\frac{l}{s_1} \\ 0 \\ 0 \\ 1 \end{array}\right\} \leftrightarrow s_0^2, \left\{\begin{array}{c} \frac{1}{l}s_1 \\ 0 \\ 0 \\ 1 \end{array}\right\} \leftrightarrow -l^2 + s_0^2 - s_1^2 \tag{4.37}$$

The equation (4.34) has trivial solution (4.30): $\boldsymbol{\alpha} = \{0,0,0\}, \varepsilon = 0$. The equation (4.34) has nontrivial solution if, at least, one of eigenvalues of the matrix (4.36) vanishes. For timelike vector $\{s_0, \mathbf{s}, l\}$ we have

$$s_0^2 > \mathbf{s}^2 + l^2 \tag{4.38}$$

Let us try to find such values of the variable l, for which the eigenvalue vanishes. The first eigenvalue of (4.37) is positive always. The second eigenvalue of (4.37) vanishes, if

$$s_0^2 - s_1^2 - l^2 = 0 \tag{4.39}$$

Fulfilment of equation (4.39) is impossible, because of (4.38). It means, that eigenvalues of the matrix (4.36) do not vanish and the equation (4.34) has only trivial solution

$$\boldsymbol{\alpha} = \{0,0,0\}, \quad \alpha_0 = 0, \quad \varepsilon = 0, \quad \alpha_5 = 0, \quad \beta = l \qquad \text{if} - L/2 < l \leq L/2 \tag{4.40}$$

Let us consider the case

$$\gamma = -2L, \qquad L < l + \beta \leq 3L \tag{4.41}$$

In the nonrelativistic case $\mathbf{s}^2, l^2, L^2 \ll s_0^2$ the equation (4.27) takes the form

$$\boldsymbol{\alpha}^2 = (l + \beta + \gamma)^2 - 2\beta^2 - 2l^2$$

$$\boldsymbol{\alpha}^2 + (l - \beta)^2 - 2L\,(2L - 2\,(l + \beta)) = 0 \tag{4.42}$$

This equation can be written in the form

$$l + \beta = -\frac{\boldsymbol{\alpha}^2 + (l - \beta)^2}{4L} + L \leq L \tag{4.43}$$

As it follows from comparison of (4.41) and of (4.43) the equation (4.42) has no solution, satisfying the inequality (4.41) even in the case when

$$\boldsymbol{\alpha} = \{0,0,0\}, \qquad l = \beta = L/2 \tag{4.44}$$

Let us consider the case

$$\gamma = 2L, \qquad -3L < l + \beta \leq -L \tag{4.45}$$

In the nonrelativistic case $\mathbf{s}^2, l^2, L^2 \ll s_0^2$ the equation (4.27) takes the form

$$\boldsymbol{\alpha}^2 = -\,(l - \beta)^2 + 2L\,(2L + 2\,(l + \beta)) \tag{4.46}$$

According with (4.45) this equation can be written in the form

$$l + \beta = \frac{\boldsymbol{\alpha}^2 + (l - \beta)^2}{4L} - L \leq -L \tag{4.47}$$

As it follows from the equation (4.47) and inequality (4.45), that the solution of equation (4.47) has the form

$$\boldsymbol{\alpha} = \{0,0,0\}, \qquad l = \beta = -L/2 \tag{4.48}$$

Uniting (4.40) with (4.48), one obtains, that if vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ are timelike in the sense (4.16), the unique solution of (4.22) is

$$\boldsymbol{\alpha} = 0, \qquad \alpha_5 = 0, \qquad \alpha_0 = 0, \qquad -\frac{L}{2} \leq l \leq \frac{L}{2} \tag{4.49}$$

Thus, one obtains, that at the point P_1 there is only one vector $\mathbf{P}_1\mathbf{P}_2 = \{s_0, s_1, s_2, s_3, l\}$, which is equivalent to the vector $\mathbf{P}_0\mathbf{P}_1 = \{s_0, s_1, s_2, s_3, l\}$ at the point P_0. This equivalence takes place only, if l satisfies the relation

$$|l| \leq \frac{L}{2} \tag{4.50}$$

If the relation (4.50) is not satisfied, at the point P_1 there is no vector $\mathbf{P}_1\mathbf{P}_2$, which is equivalent to the vector $\mathbf{P}_0\mathbf{P}_1 = \{s_0, s_1, s_2, s_3, l\}$.

If the Kaluza-Klein geometry is not compactificated the vectors $\mathbf{P}_0\mathbf{P}_1 = \{s_0, s_1, s_2, s_3, l\}$ and $\mathbf{P}_1\mathbf{P}_2 = \{s_0, s_1, s_2, s_3, l\}$ are equivalent at any value of the charge l (at $l^2 < s_0^2 - \mathbf{s}^2$). Thus, the compactification discriminates large values of the charge coordinate $x^5 = l$. Influence of compactification reminds influence of a potential hole with infinite high walls placed at the values $-L/2$ and $L/2$ of the fifth coordinate x^5. In both cases displacement of a particle in the fifth direction is restricted. In the case of the potential hole only displacement (but not momentum p_5) is restricted. In the case of compactification the value of momentum p_5 (electric charge) is restricted, when the physical space-time geometry is used. In this case the links of the world chain have finite length and a discrimination of large values of the electric charge appears. At the conventional approach to the Kaluza-Klein geometry, based on the linear vector space, the compactification does not discriminate any values of the electric charge (in the case of classical dynamics), because the length of the particle world chain links is considered to be infinitesimal.

The charge coordinate $x^5 = l$ describes a displacement of the particle in the fifth direction x^5. In the physical geometry the component $x^5 = l$ of the vector $\mathbf{P}_0\mathbf{P}_1 = \{s_0, 0, 0, 0, l\}$, is simultaneously a component p_5 of the momentum vector (the electric charge to within a factor). The discrimination of values of the quantity l is a discrimination of the charge component p_5 of the momentum vector $\mathbf{P}_0\mathbf{P}_1$, i.e. it is a discrimination of the particle electric charge. Conventional method of compactification, using a single-valued metric tensor (but a many-valued world function), does not admit one to obtain a restriction of the module of the electric charge of an elementary particle. It generates only periodical dependence of the particle state on the fifth coordinate. Of course, this periodical dependence relates only to the state of the statistical ensemble, but not to the state of a single particle. Instead of the dynamic equations for a single free particle

$$\frac{d}{dt}x^A(t) = v^A(t), \qquad \frac{dv^A(t)}{dt} = 0, \qquad A = 1,2,3,5 \tag{4.51}$$

one should consider dynamic equations for the statistical ensemble consisting of free particles, whose motion is described by equations (4.51). In particular, if the state of this ensemble is described by the wave function $\psi(t, \mathbf{x}, x^5)$, the wave function is to be a periodical function of the fifth coordinate x^5

$$\psi\left(t, \mathbf{x}, x^5\right) = \psi\left(t, \mathbf{x}, x^5 + 2kL\right) \tag{4.52}$$

where k is any integer number. In the framework of quantum mechanics this periodicity leads to that result, that the operator of the electric charge $-i\hbar\partial/\partial x^5$ has eigenvalues which are multiple to some elementary electric charge.

After compactification the single-valued world function restricts the particle displacement in the direction of fifth coordinate. However, in general, it does not restrict the charge component p_5 of the momentum vector. The the charge component p_5 is restricted, if (1) *the links of the world chain have a finite length* and (2) *the world function is single-valued.* If one of these conditions is violated, the value of the charge component p_5 of the momentum vector may be not restricted.

In particular, at the conventional approach, when the world function becomes to be many-valued after compactification, the particle displacement along the direction x^5, and momentum p_5 of a particle remain to be unrestricted. If the world function is single-valued after compactification, but the world chain links are infinitesimal, the particle displacement along the direction x^5 appears to be restricted, but the momentum p_5 remains to be unrestricted. In particular, in the discrete space-time geometry, where the links of the world chain cannot be infinitesimal, the momentum p_5 appears to be restricted, if the world function is made single-valued after compactification.

It is well known, that stable elementary particles have the electric charge $0, \pm e_0$, where e_0 is the elementary charge. Only short-living resonances have multiple charges. Apparently, they are bound states of several elementary particles. As to quarks, which have fractional electric charge, they cannot be extracted from stable elementary particles. Quarks are rather elements of a structure of elementary particles, than elementary particles themselves.

Thus, experimental data confirm a reasonable supposition on single-valuedness of the world function after compactification.

5 Compactification in the discrete Kaluza-Klein space-time

Let us consider compactification of the discrete Kaluza-Klein space-time. The world function has the form

$$\sigma_{\mathrm{dK}}\left(x, x'\right) = \sigma_{\mathrm{K}}\left(x, x'\right) + \frac{\lambda_0^2}{2}\mathrm{sgn}\left(\sigma_{\mathrm{K}}\left(x, x'\right)\right) \tag{5.1}$$

where σ_{K} is determined by the relation (4.3), (4.4). As far as the space-time geometry with the world function (5.1) is discrete, the world chains of particles have links of a finite length, because in the discrete space-time geometry the link length cannot be infinitesimal. We consider two timelike vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ of the world chain. The vectors are determined by the relations (4.8) - (4.12). These vectors are supposed to be equivalent and to satisfy the relations of the type (4.14), (4.15).

The equations are rewritten in the developed form

$$s_0^2 - \mathbf{s}^2 - l^2 = (s_0 + \alpha_0)^2 - (\mathbf{s} + \boldsymbol{\alpha})^2 - (l + \alpha_5)_{\rm st}^2 \tag{5.2}$$

$$\left((2s_0 + \alpha_0)^2 - (2\mathbf{s} + \boldsymbol{\alpha})^2 - (2l + \alpha_5)_{\rm st}^2\right) + \lambda_0^2 = 4\left(\left(s_0^2 - \mathbf{s}^2 - l^2\right) + \lambda_0^2\right) \tag{5.3}$$

Combining equations (5.2) (5.3), one obtains

$$\alpha_0 = \frac{2\mathbf{s}\boldsymbol{\alpha} + (2l + \alpha_5)_{\rm st}^2 - (l + \alpha_5)_{\rm st}^2 - 3l^2 + 3\lambda_0^2}{2s_0} \tag{5.4}$$

Substituting (5.4) in (5.2), one obtains

$$\boldsymbol{\alpha}^2 = (2l + \alpha_5)_{\rm st}^2 - 2(l + \alpha_5)_{\rm st}^2 - 2l^2 + 3\lambda_0^2 + \left(\frac{2\mathbf{s}\boldsymbol{\alpha} + (2l + \alpha_5)_{\rm st}^2 - (l + \alpha_5)_{\rm st}^2 - 3l^2 + 3\lambda_0^2}{2s_0}\right)^2 \tag{5.5}$$

Or

$$\boldsymbol{\alpha}^2 = (l + \gamma)(l + 2\beta + \gamma) - \beta^2 - 2l^2 + 3\lambda_0^2 + \left(\frac{2\mathbf{s}\boldsymbol{\alpha} + (l + \gamma)(l + 2\beta + \gamma) - 3l^2 + 3\lambda_0^2}{2s_0}\right)^2 \tag{5.6}$$

where γ is determined by (4.25)

We shall consider only nonrelativistic case $\mathbf{s}^2, l^2, L^2 \ll s_0^2$ for timelike vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$. In the case, when $\gamma = 0$ and according to (4.23), (4.25)

$$-L < l + \beta \leq L \tag{5.7}$$

one obtains from (5.6)

$$\boldsymbol{\alpha}^2 + (\beta - l)^2 = r_1^2, \qquad r_1^2 = 3\lambda_0^2 \tag{5.8}$$

The relation (5.8) describes a sphere of the radius

$$r_1 = \sqrt{3}\lambda_0 \tag{5.9}$$

with the center $\{\boldsymbol{\alpha}_{\rm c}, \beta_{\rm c}\} = \{\mathbf{0}, l\}$ in the 4-dimensional space of coordinates $\{\boldsymbol{\alpha}, \beta\} = \{\alpha_1, \alpha_2, \alpha_3, \beta\}$.

Solution of equations (5.8) has the form

$$\alpha_1 = r_1 \sin\theta \sin\phi_2 \sin\phi_3, \qquad \alpha_2 = r_1 \sin\theta \sin\phi_2 \cos\phi_3, \tag{5.10}$$

$$\alpha_3 = r_1 \sin\theta \cos\phi_2, \qquad \beta = l + r_1 \cos\theta, \qquad \alpha_0 = 0, \tag{5.11}$$

which is valid for

$$r_1 \cos\theta \leq L - 2l. \tag{5.12}$$

Here r_1 is defined by relation (5.9), and θ, ϕ_2, ϕ_3 are arbitrary numbers. Although the solution (5.10), (5.11) is many-valued, but it is placed at the distance of the order of λ_0 from the single-valued solution (4.49). At $\lambda_0 = 0$ the radius r_1 of the sphere (5.8) vanishes and the solution (5.10), (5.11) coincides with (4.49).

We are interested in behaviour of the solutions the equation (5.8) near the boundary $x^5 = L/2$. The sphere (5.8) lies either completely inside the region $-L/2 < x^5 < L/2$, or intersects the boundary $x^5 = L/2$. In the last case we use the designation

$$l = \frac{L}{2} + \delta, \qquad -\frac{r_1}{2} < \delta \leq \frac{r_1}{2} \tag{5.13}$$

then according to (5.11)

$$\beta = \frac{L}{2} + \delta + r_1 \cos\theta \tag{5.14}$$

The angle $\theta_{\max}$ of intersection of the sphere (5.8) with the boundary $x^5 = L/2$ is defined by the relation

$$\cos\theta_{\max} = -\frac{2\delta}{r_1} \tag{5.15}$$

which follows from (5.12). Position of the sphere section is determined by

$$\beta_{\max} = L/2 + \delta + r_1 \cos\theta_{\max} = L/2 - \delta \tag{5.16}$$

which corresponds to the condition

$$\beta_{\max} + l = L \tag{5.17}$$

Radius R of the sphere section has the form

$$R = r_1 \sin\theta_{\max} = r_1\sqrt{(1 - \cos^2\theta_{\max})} = \sqrt{r_1^2 - 4\delta^2} \tag{5.18}$$

One can see, that the value $x^5_{(2)} = \beta$ of the charge coordinate x^5 of vector $\mathbf{P}_1\mathbf{P}_2$ is always less, than $\frac{L}{2} + \frac{r_1}{2}$. Besides, the value $x^5_{(2)} = \beta$ of the charge coordinate $x^5_{(2)}$ of vector $\mathbf{P}_1\mathbf{P}_2$ is less, than $L/2$, if the charge coordinate $x^5_{(1)} = l$ of vector $\mathbf{P}_0\mathbf{P}_1$ is larger, than $L/2$. In other words, the charge coordinate x^5 "reflects itself" from the boundary $x^5 = L/2$ in the following sense. If the coordinate $x^5_{(1)}$ of the vector $\mathbf{P}_0\mathbf{P}_1$ appears near the boundary $x^5 = L/2$, $x^5_{(1)} \in (L/2 - r_1/2, L/2)$, then coordinate $x^5_{(2)}$ of the vector $\mathbf{P}_1\mathbf{P}_2$ may be larger than $L/2$. For instance, it is possible, that $x^5_{(2)} \in (L/2, L/2 + r_1/2)$. However, coordinate $x^5_{(3)}$ of the next link $\mathbf{P}_2\mathbf{P}_3$ will be less than $L/2$. According to (5.14) $x^5_{(3)} \in \left(\frac{L}{2} - \frac{3r_1}{2}, \frac{L}{2} - \frac{r_1}{2}\right)$. Thus, the world chain cannot go through the boundary $x^5 = L/2$, although single points of the world chain may have coordinate $x^5 \in (L/2 + r_1/2)$. Behaviour of the world chain near the boundary $x^5 = -L/2$ is the same, as near the boundary $x^5 = L/2$. The world chain reflects itself from the boundary $x^5 = -L/2$. Thus, the world chain will be placed in the region $-L/2 < x^5 < L/2$.

In the case of continuous space-time, when $\lambda_0 = 0$, the world chain does not penetrate through the boundary $x^5 = L/2$. In the case of the discrete space-time, when $\lambda_0 > 0$, one point of the world chain may penetrate through the boundary $x^5 = L/2$. However, the next point of the world chain returns to the region $-L/2 < x^5 < L/2$. We shall refer to solutions, satisfying the condition (5.7), as basic solutions, whereas the solutions, satisfying conditions (4.41), as additional solutions. Behaviour of the world chain near the boundary $x^5 = L/2$ reminds behavior of a quantum particle near the wall of the potential hole.

Let us consider the case (additional solutions), when

$$\gamma = -2L, \qquad L < l + \beta \leq 2L \tag{5.19}$$

In the nonrelativistic case the equation (5.6) takes the form

$$\boldsymbol{\alpha}^2 + (\beta - l + 2L)^2 = r_2^2, \qquad r_2 = \sqrt{8L^2 - 8lL + 3\lambda_0^2} \tag{5.20}$$

This equation describes the sphere of radius r_2 with center at the point $\{\boldsymbol{\alpha}_c\beta_c\} = \{\mathbf{0}, l - 2L\}$. It follows from (5.20) that

$$\begin{aligned}\alpha_1 &= r_2 \sin\theta_{\rm ad} \sin\phi_2 \sin\phi_3, \qquad \alpha_2 = r_2 \sin\theta_{\rm ad} \sin\phi_2 \cos\phi_3, \qquad \alpha_3 = r_2 \sin\theta_{\rm ad} \cos\phi_2, \\ \beta &= l - 2L + r_2 \cos\theta_{\rm ad}, \qquad \alpha_0 = 0, \qquad \text{for} \quad r_2 \cos\theta_{\rm ad} \geq 3L - 2l \end{aligned} \tag{5.21}$$

where $\theta_{\rm ad}, \phi_2, \phi_3$ are arbitrary values. In the case (5.19) and $\lambda_0 = 0$ we obtain, that $r_2 = 2L$ $\beta = l = L/2$.

Let us set

$$l = L/2 + \delta, \qquad |\delta| \leq \frac{r_1}{2} \tag{5.22}$$

One obtains from the first inequality (5.19) and (5.21)

$$0 < 2\delta - 2L + r_2 \cos\theta_{\rm ad} \tag{5.23}$$

where $\theta_{\rm ad}$ is the angle between the axis and generatrix of the cone with vertex at the point $(\boldsymbol{\alpha}_c\beta_c) = (\mathbf{0}, l - 2L)$. The cone is based on the set of solutions on the sphere (5.20). One obtains from (5.18) and (5.19) for the minimal value $\cos\theta_{\rm min}$ of the quantity $\cos\theta_{\rm ad}$

$$\cos^2\theta_{\rm min} = \frac{(2L - 2\delta)^2}{4L^2 - 8L\delta + r_1^2} \tag{5.24}$$

$$\sin^2\theta_0 = 1 - \cos^2\theta_{\rm min} = \frac{\frac{1}{4}r_1^2 - \delta^2}{r_2^2}, \qquad \delta^2 \leq \frac{1}{4}r_1^2 \tag{5.25}$$

It follows from (5.25) that in the case (5.19) there are additional solutions, if

$$l = L/2 + \delta, \qquad \delta^2 < \frac{r_1^2}{4} \leq \frac{L^2}{4} \tag{5.26}$$

The minimal value $\beta_{\rm min}$ is determined by the condition

$$\beta_{\rm min} + l = L \tag{5.27}$$

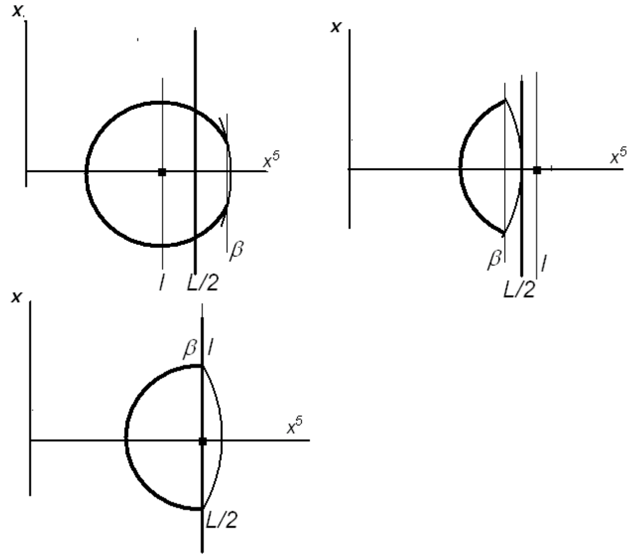

Figure 1: Set of solutions, depending on the value of l, shown by small dark square. Usual solutions are shown by thick line. Additional solutions are shown by thin line.

which coincides with the maximal value (5.17) for basic solutions. Radius $R_{\rm ad}$ of the corresponding section of the sphere (5.20) has the form

$$R_{\rm ad} = r_2 \sin\theta_0 = \sqrt{r_1^2 - 4\delta^2} \qquad (5.28)$$

which coincides with the radius (5.18). The solutions are shown in figure 1

Action of the boundary $x^5 = L/2$ changes the surface of the sphere of basic solutions. The set of all solutions has a shape of two connected spherical segments of different radius. Restriction (4.50) on the electric charge of a particle is connected directly with a finite length of the world chain links. In the discrete space-time geometry (5.1) the length of the world chain link is finite by necessity (but not infinitesimal), because the infinitesimal length does not exist. In the continuous space-time geometry, the link of the world chain may be infinitesimal, in principle. In this case the space-time compactification does not restrict the maximal value of the electric charge.

Experimental data show, that the electric charge of a stable elementary particle is equal to $0, \pm e_0$, where e_0 is the elementary charge of an elementary particle.

6 Restriction on maximal charge of elementary particle

Thus, a compactification of the Kaluza-Klein space-time leads to a discrimination of some values p_5 of the charge momentum component. The discrimination is a corollary of the fact, that not all links of the world chain of a particle are possible in the compactified geometry. In the conventional approach to the Kaluza-Klein geometry, when the space-time geometry is constructed as a Riemannian geometry (but not as a physical geometry), there is no discrimination of the maximal value of the momentum component p_5.

In all cases the electric charge has the form $e = ne_0$, where e_0 is the elementary charge and n is an integer number. This fact is connected with the periodicity of the wave function with respect to fifth coordinate x^5. Averaging random world chains, one obtains a dynamic equation of the type of the Schrödinger equation. In the region, where the point P_2 of the vector $\mathbf{P}_1\mathbf{P}_2$ is placed on the sphere (5.8) this equation has the form

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_5}\frac{\partial^2\psi}{\partial\left(x^5\right)^2} - \frac{\hbar^2}{2m_5}\mathbf{\nabla}^2\psi \tag{6.1}$$

In other regions, where the set of solutions is not a sphere, this equation is modified. It takes a form of the Schrödinger equation in a potential hole with walls at $x^5 = \pm L/2$. We shall not to take into account this modification and use the equation (6.1) for approximate estimation of connection between the period $2L$ and possible values of the of elementary length λ_0.

Solution of equation (6.1) has the form

$$\psi\left(t,\mathbf{x},x^5\right) = \sum_n a_n \exp\left(-\frac{i}{\hbar}E_n t + i\frac{\mathbf{p}_n\mathbf{x}}{\hbar} + i\frac{(p_5)_n}{\hbar}x^5\right) \tag{6.2}$$

where

$$\left(p_5\right)_n = \frac{e}{\varkappa c}n, \qquad E_n = c\sqrt{m_5^2c^2 + p_n^2 + \left(p_5\right)_n^2} \tag{6.3}$$

n is an integer number, $\varkappa$ is some universal constant (3.2). a_n are arbitrary complex numbers. The quantity $m_5 = \text{const}$ is a 5-mass of the particle, whereas the usual mass $m = \sqrt{m_5^2 + p_n^2c^{-2}}$ depends on the fifth component p_5 of momentum . In order that ψ be a single-valued function of x^5, the momentum p_5 is to have the form

$$\left(p_5\right)_n = \frac{\pi\hbar}{L}k_n, \qquad k_n \quad \text{is} \quad \text{integer} \tag{6.4}$$

The wave function (6.2) must describe a stationary state of the particle, because in a nonstationary state the charged particle, placed in a potential hole, radiates electromagnetic waves. As a result the particle appears very rapidly at a stationary state, where the charge density and charge current are constant, and the particle ceases to radiate. The wave function (6.2) is single-valued, if all E_n in the sum (6.2) are equal, and any E_n is not changed at a variation of k_n. These conditions are fulfilled, if the sum (6.2) contains only one term, and the momentum has the form

$$p_5 = \frac{\pi\hbar}{L}s \qquad \left(p_5\right)_n = \frac{e}{\varkappa c}n \tag{6.5}$$

where s is some definite integer number.

It follows from comparison of relations (6.3) and (6.5), that

$$\varkappa = \frac{e_0 L}{\pi \hbar c} \tag{6.6}$$

where e_0 is the elementary electrical charge, and $2L$ is the period of the fifth coordinate x^5.

On the other hand, the momentum p_5 along the fifth direction is connected with the geometrical momentum π_5 by means of the relation

$$p_5 = bc\pi_5 \tag{6.7}$$

According to the relation (4.50), where the universal constant b is defined by the relation (2.5),

$$|\pi_5| = |l| < \frac{L}{2}, \tag{6.8}$$

Using relations (6.5), (6.7), (6.8), one obtains

$$\pi_5 = \frac{p_5}{bc} = \frac{1}{bc}\frac{\pi\hbar}{L}s = \pi\frac{\lambda_0^2}{L}s \tag{6.9}$$

and

$$|s| < \frac{L^2}{2\pi\lambda_0^2} \tag{6.10}$$

Taking maximal value $r_1 = \sqrt{3}\lambda_0 = L/2$, one obtains

$$|s| < \frac{3L^2 4}{2\pi L^2} = 1.909\,9 \tag{6.11}$$

$|s| = 1$, and the module $|e|$ of the charge e of a stable elementary particle is not more than the elementary charge e_0. In general, the approximation (6.1) is too rough, and the relation (6.11) may not be considered as a true relation. Firstly we have used the approximate equation (6.2). Secondly, the choice $r_1 = L/2$ is not founded exactly. However, it is important, that in any case the maximal electric charge of a stable elementary particle is restricted. The exact value of s may be obtained at at proper choice of λ_0. The relation (6.11) shows, that if the period $2L$ of the fifth coordinate x^5 is of the order of λ_0, it is possible such an interrelation between L and λ_0, that the module $|e|$ of the charge e of a stable elementary particle is not more, than the elementary charge e_0. It is essential, that the restriction has a geometrical form. It connects the elementary length λ_0 with the length $2L$ of compactification.

Thus, compactification of the Kaluza-Klein space-time geometry imposes restrictions on possible values of the electric charge of an elementary particle. One needs only to use the physical geometry, which uses uniform formalism for description of continuous and discrete geometries.

7 Concluding remarks

Discreteness of the space-time in microcosm seems to be a more simple and reasonable supposition, than the opposite supposition on continuous space-time equipped by quantum principles. Discreteness of the space-time admits one to describe quantum effects without referring to quantum principles. Describing discreteness of the space-time, the elementary length λ_0 determines the quantum constant $\hbar$. The space-time discreteness appears to be compatible with its isotropy and its uniformity. However, this compatibility can be understood only in framework of physical geometry, which uses the same formalism for description of discrete and continuous geometries. Combination of the discrete space-time with its compactification admits one to obtain restrictions on the electric charge of stable elementary particles. These restrictions are known from experiments, but they have no explanation in the framework of the conventional quantum theory.

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