Author's comments to referee's reports on the paper by Y. A. Rylov "Dynamical methods of investigation in application to the Dirac particle", submitted to a scientific journal

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Abstract

The paper presents a discussion between the author and the referee. The discussion is devoted essentially to the question, whether or not the Dirac particle has internal degrees of freedom, described nonrelativistically. The author's viewpoint is based on the radical approach, which may be described as the third modification of the space-time geometry (the first modification is the special relativity, the second one is the general relativity). The third modification generates purely dynamical methods of investigation, which are free of quantum principles, because they are not needed. On the contrary, the referee's viewpoint is the conventional one, based on the quantum principles. Application of the two different methods to the investigation of the Dirac particle is discussed.

1 Introduction

In this paper we present comments to the referee's reports on the paper [1], submitted to a scientific journal. What kind of a strange scientific publication is such a paper? Why publish the referee's reports and comments to them, if it is sufficient to publish the paper in itself?

To answer these puzzling questions, we note that the paper [1] is a pioneer paper, which uses unexpected dynamical methods of investigation of the well known dynamic system $\mathcal{S}_{\mathrm{D}}$, described the Dirac equation. This dynamic system $\mathcal{S}_{\mathrm{D}}$ will be referred to as the Dirac particle. In general, the dynamical methods of investigations are not new. They are reasonable methods, which are used at investigation of any dynamic system. However, the Dirac particle is not simply a dynamic system. It is a quantum dynamic system, which is not investigated by purely dynamical methods. It is always investigated by quantum methods, i.e. by the dynamic methods, **constrained by the quantum principles**. Results of investigations by quantum methods and by the purely dynamic methods may appear to be different, because any physical theory is essentially a totality of mathematical methods, which it uses.

The purely dynamic methods of the quantum systems investigation are generated by some new approach to a description of quantum phenomena. This approach may be defined as the third modification of the space-time geometry [2] (the first modification is the special relativity, the second one is the general relativity). The third modification is rather unexpected and radical. In the modified space-time geometry the motion of a free particle is primordially stochastic, and intensity of its stochasticity depends on the particle mass. At the same time the modified space-time geometry is a geometry in the exact sense of this word. This geometry is described completely by the world function, as the Euclidean geometry and the Minkowski geometry are described. It is distinguishes essentially from stochastic geometries and quantum geometries, which are not geometries in the exact sense of this word, because they are the usual Minkowski geometry, provided by additional structures (such as probabilistic structures, or matrices, describing non-commutative quantities).

The third modification of the space-time geometry generates a new dynamics, which describes dynamic systems and stochastic ones by means of the same united formalism [3]. Ignoring completely quantum principles, the united formalism uses the classical principles of dynamics and works in the framework of the classical dynamics. In general, the united formalism of dynamics may work without a reference to the third modification of the space-time geometry, because it works essentially in the Minkowski space-time, taking from the third modification only the primordial stochasticity of the free particle motion. However, the united formalism of dynamics can be constructed only on the basis of the dynamic conception of the statistical description. It cannot be constructed, if we believe, that the statistical description and the probabilistic description is the same, and that the probability theory is a necessary component of any statistical description.

Being applied for investigation of the Dirac particle, the united formalism of dynamics leads to the unexpected results, that the Dirac particle has internal degrees of freedom [4] and these internal degrees of freedom are described nonrelativistically [5]. The average researcher cannot trust in such results, because it is written in all textbooks that the Dirac particle is pointlike and relativistic. The average researcher ignores simply the papers like [4], [5], because there is a lot of wrong papers, and to read them is a loss of time. At first, the nonrelativistical character of the internal degrees of freedom for the Dirac equation was obtained ten years ago [6]. Although this result is very important for further development of the relativistic quantum theory, where the Dirac equation plays an important role, this result has been ignored in the last ten years. Such a situation is rather characteristic at critical points of the physics development.

Now the paper [1] is submitted to a scientific journal (we do not mention its title because this title is a confidential factor of correspondence with its editor). The editor of this journal appears to be scrupulous, and he sends the submitted paper to the referee. (note, that unscrupulous editor rejects usually such a radical paper at once, because it is difficult to find a qualified referee for such questionable (from the editor's viewpoint) papers). The referee appeared to be also scrupulous, and he (she) was forced to write a scrupulous report. The referee is the average researcher, and he (she) disagrees with the nonrelativistic character of the internal degrees of freedom for the Dirac particle. The paper has a theoretical character, and the referee cannot disprove the results, referring to experimental data. The referee was forced to look for mistakes in the submitted paper, because it was the only way to disprove results scrupulously (there is a lot of ways to make this unscrupulously). As a result the scrupulous referee is forced to adduce arguments against results, obtained in the submitted paper. These arguments are arguments of an average researcher, and they are clear for any researcher. The author of the submitted paper can disprove these arguments, but he cannot invent them, because these arguments are outside of his approach to the investigation of the Dirac particle. As a result we obtain an interesting discussion, which helps us to approach to the truth. The goal of the present paper is explanation of the new approach for the average researcher in the example of the discussion between the referee and the author. We hope also, that this paper will be useful for advanced researchers, who have no time for a reading of questionable papers.

The two following section contain the text of reports (in Roman) and the author's comments to them (in Italics). Unfortunately, the discussion is not finished, because the editorial policy of the journal forbids a consideration of more than two negative reports.

2 Comments to the first report

Referee:

In the paper the author studies, in particular, the nonrelativistic approximation of the Dirac equation. He claims that in this approximation the Dirac equation admits "high frequency solutions" which have been lost. As the author argues, the reason of this loss is in neglecting the highest derivatives which is a mathematical mistake. This mistake, in turn, appeared (in the author opinion) due to the "experimental-fitting methods" which dominate in the modern physics. In contrast to experimental-fitting methods the author calls to use the Newtonian method with the slogan "Hypotheses non fingo".

I find that it is very hard to argue with the author. The reason of this is not that author's investigation is perfect, but rather that the author sometimes contradicts himself. For example, it is written after Eq. (5.3) that high frequency solution is associated with the nonrelativistic antiparticle. At the same time, it is written in the introduction that "one cannot neglect the terms (which lead to the high frequency solution – my comment), which are connected with the internal structure of the Dirac particle". I could ask the author to clarify to himself what high frequency solution corresponds to: the antiparticle or the internal structure of a particle.

Author's comment:

The internal structure of the Dirac particle is connected with the high frequency solution in the following sense. From the dynamical viewpoint the Dirac particle is a rotator, consisting of two subparticles rotating with the frequency of the order $\Omega = 2mc^2/\hbar$ *around their inertia center. The frequency* Ω *describes the rigidity of the internal structure (rotation). On the other hand, the same frequency* Ω *is the characteristic frequency of the high frequency solutions. If the low frequency solutions describe the Dirac particle, the high frequency solutions (with the characteristic frequency* Ω*) are associated with some alternative solutions (Dirac antiparticle). The high frequency solutions are only associated with the antiparticles (but not describe them), because the energy of the high frequency solutions is negative and the charge has the same sign, as the low frequency solutions have. The negative energy of the high frequency solutions is a defect of the dynamic system. There may be different ways of removing this defect and different ways of a connection between the high frequency solutions and the antiparticle description. But I do not see* ***any contradiction*** *between the fact that, on the one hand, the frequency* Ω *is connected with the internal structure and, on the other hand, it is associated with the antiparticle description.*

Referee:

Another example is the introduction of the last term in Eq. (2.9), as well as the procedure of the "dynamic disquantization", in particular Eq.(3.5). It seems that both these approaches contradict to the Newtonian slogan to the same degree as "the axiomatic representation of the quantum mechanics" does.

Author's comment:

There is no contradiction between consideration of Eq. (2.9) and the Newtonian investigation strategy. The Newtonian slogan relates only to change of principles of dynamics, but not to construction of Lagrangians and dynamic equations for concrete physical systems. For instance, if one describes a pendulum without friction, one describe it by a dynamic equation. If one describes the pendulum with a friction, one adds to the dynamic equation a term, which is responsible for the vibration damping. This additional term is not a hypothesis, because the principles of dynamics are the same for the ideal pendulum and for the pendulum with the damping.

If we transit from the free classical particle to the quantum one, we say that we should replace the Hamilton function by the Hamilton operator, replacing momentum p *by operator* $-i\hbar\nabla$*. In this case we change classical principles of dynamics and violate the Newtonian slogan. When we transit from the statistical ensemble of classical particles, described by the action (2.12), to the quantum (stochastic) particle, described by the action (2.9), we change only Lagrangian of the dynamic system, that is quite natural. But the principles of classical dynamics remains. Hence, at this point the Newtonian slogan is not violated. In the given case the Newtonian* *slogan means:* ***There is no necessity to change principles of dynamics, if the same result can be obtained by a change of Lagrangian****. In general, the validity of the dynamical principles is verified by a test of experimental data relating to all physical phenomena, whereas the validity of the Lagrangian is verified by a test of experimental data relating to the given dynamic system.*

As to the dynamic disquantization (3.5), it is simply a method of the dynamic system investigation. It does not deal with dynamics in itself. This method admits one to obtain the system of ordinary differential equation from the system of partial differential equations. It is relativistical procedure, which admits one to obtain the system of ODE uniquely, provided the current vector j^k of the dynamic system is given. The dynamic disquantization has no relation to the Newtonian slogan, because it is not dynamics, but only a method of the dynamics investigation. One may use, or not use this method. Dynamic equations generated by the action (2.12) are ODE, whereas dynamic equations, generated by the action (2.9) are PDE, and the last term in (2.9) is responsible for transition ODE→PDE. The dynamic disquantization transforms PDE in ODE, and at the correctly chosen j^k it is equivalent to elimination of the last term of (2.9). All procedures are dynamical, and principles of classical dynamics remain.

Referee:

Nevertheless, I let me try "to discover mistakes in the author investigations".

Mistakes:

1) The author thinks that there is a mathematical mistake in the transition to the nonrelativistic approximation and high frequency solutions have been lost. This is not the case, however. There are several people who knew that the general solution to (4.17) is (4.25). Fortunately, they did not keep this fact a secret and this result has been published. See, for example, the following well-known textbooks and refs. therein: J.D. Bjorken, S.D. Drell, Relativistic Quantum Field Theory; S.Weinberg, The Quantum Theory of Fields; B.Thaller, The Dirac Equation. These studies, however, refer to this type of solutions not as "high frequency solutions" but rather as the NEGATIVE ENERGY SOLUTIONS. The literal coincidence of these negative energy solutions with those the author found follows from the substitution of (4.25) into (4.7). The reason of why these solutions are usually discarded when one looks for nonrelativistic approximation of the Dirac equation, is hiding not in a mathematical mistake but in the conceptual difficulties of the ONE-particle nonrelativistic quantum mechanics which cannot interpret states with spectrum unbounded from below. This issue is resolved in the framework of quantum field theory (QFT) where Dirac wave function changes its meaning and becomes an operator. Another approach, which may look now antique, is the theory of the Dirac holes. In any case, the wave function becomes MULTIPARTICLE, i.e. it describes multiparticle states (for the details see the textbooks pointed out above).

Author's comment:

As far as I could understand from this passage the referee suggest that I have made a mistake, when I stated that the high frequency terms should be taken into account in the nonrelativistic approximation of the Dirac equation. The referee states *that the high frequency terms should be removed, because they have negative energy.*

The referee is quite right, that the high frequency solutions are the negative energy solutions, and existence of these states is a defect of dynamic system (4.1). However, I suppose that investigating dynamic system, one should investigate it properly independently of whether or not this dynamic system has defects. In general, if we see defects in the dynamic system, we should change the dynamic system and construct such a dynamic system, which has no defects. However, the method of investigation must be the same for the correct dynamic systems and for the defective ones. This method states that the high frequency solutions should be remained in the nonrelativistic approximation. But there are other reasons, according to which these terms may be omitted. I suggest that the high frequency solutions together with the low frequency solutions are unstable (top paragraph of p.24) and they must disappear as a result of electromagnetic emanation. The referee supposes, that the high frequency solutions should be omitted, because they are the negative energy solutions. Where is my mistake? Conventional arguments in favour of elimination of the high frequency solutions and my arguments are different. Is it my mistake?

I consider that defects of dynamic system should be eliminated by a modification of the dynamic system. At the conventional approach one believes that the defects can be removed in the framework of the same defective dynamic system by a change of interpretation. This is the difference between the two approaches.

At the conventional approach one does not remove defect of the dynamic system (4.1). Instead, one tries to remove undesirable corollaries of the defect by means of non-dynamical methods. One uses quantum principles (changing the dynamic variable ψ by an operator $\hat{\psi}$), which determine some interpretation of dynamic variables and of dynamic system. Conventional interpretation of the solutions is a non-dynamical procedure, which is produced outside the framework of principles of classical dynamics, and one should test compatibility of interpretation with dynamic equations.

It means mathematically that the commutation relations imposed on the operator $\hat{\psi}$ are to be compatible with the dynamic equations. Unfortunately, as far as I know, nobody tests this compatibility, supposing that the commutation relations are something like the initial data, which may be given in arbitrary way. I have not tested compatibility of commutation relations in the case of the Dirac particle, but I have tested this in the case of the Klein-Gordon equation (hep-th/0106169). Result is as follows. The conventional simultaneous commutation relations

$$\left[\varphi\left(0,\mathbf{x}\right),\varphi^{+}\left(0,x'\right)\right]_{-}=\delta\left(\mathbf{x}-\mathbf{x}'\right)$$

for the complex nonlinear Klein-Gordon equation

$$\partial_k\partial^k\varphi+m^2\varphi=\lambda\varphi^{+}\varphi\varphi,\qquad\lambda=\mathrm{const}$$

are compatible with the dynamic equation only at $\lambda=0$, i.e. only for the linear equation. (In particular, the incompatibility displays itself, in the nonstationarity of the vacuum state.) I believe that something like that may take place for the Dirac *particle in the electromagnetic field. At any rate, anybody, who uses conventional anticommutation relations for the Dirac wave function* $\hat{\psi}$*, must prove their compatibility with the Dirac equation at nonvanishing electromagnetic field. (I am not sure that they are compatible, but I am not going to test this. It is the business of researchers, who use the secondary quantization). I prefer to investigate the Dirac particle as a dynamic system by means of dynamic methods and not to mix the dynamics with interpretation. In this case the investigation is not restricted by the form of interpretation (quantum principles).*

Referee:

2) The author uses the ONE-particle interpretation of the Dirac wave function which is an essential part of the procedure of "dynamic disquantization". Indeed, the main role in this procedure is played by the current. So, to obtain classical approximation of ONE Dirac particle, one needs ONE-particle current.

Author's comment:

It is misunderstanding. I do not use the ONE-particle interpretation, I do not use any interpretation at all (I mean that interpretation is something additional to the concept of the dynamical system). It is true, that the current j^k *is essential for the procedure of dynamic disquantization. But why the referee has decided, that the current is the one-particle current. Because that the dynamical variable* $\psi = \psi(x)$ *is a function of one space-time point? But the wave function* $\psi = \psi(x)$ *of one argument* x *may describe by means of the current (4.4) the state of one world line. In turn, making zigzags in time direction, one world line may describe several particles and antiparticles. Besides, description may be inclusive, when the wave function* $\psi = \psi(x)$ *of one argument* x *describes not one and only one world line, but at least one world line (and, maybe, some other world lines). Thus, the current may be the current of many particles and antiparticles. The only dynamical constraint is that the total current of all particles and antiparticles is described by Eq.(4.4).*

Referee continues:

Well, saying that the QFT approach (being a product of "experimentall-fitting" method) is incorrect, the author may try to use one-particle interpretation. But in this case one essential question appears: how one can interpret negative energy (Eq.(5.6))? Another question (excuse my experimental-fitting approach) is what is the internal structure of the electron which nobody sees in experiments? Again (as in Ref[2]) the author may take the point of view that the Dirac equation has nothing in common with the correct (unknown) description of the electron. Now I agree with the author: indeed, the Dirac equation TOGETHER with one-particle interpretation and, hence, with the method of the dynamic disquantization have nothing in common with reality. I would like only to stress, that on the other hand, the Dirac equation TOGETHER with the QFT approach perfectly describe dynamics of the electrons and positrons. The simplest logical calculations show that it follows from these two statements, that the Dirac equation is TRUE, the one-particle interpretation and, hence, the dynamic disquantization is FALSE. If the author has another opinion then the author uses some new unknown logic.

Author's comment:

The referee continues to argue against one-particle interpretation, which is not used in my paper. In reality I try to investigate properly the Dirac particle with all its defects by means of only dynamic methods WITHOUT ANY ADDITIONAL CONSTRAINTS. We can hope to overcome defects of the Dirac particle (negative energy, nonrelativistic character of internal degrees of freedom) only, if we investigate properties of the Dirac particle in itself as a dynamic system. I believe, that we shall modify the Dirac particle as a dynamic system. Correction of defects on the interpretation level by means of the quantum principles seems to me unreliable, because, first, the quantum principles are nonrelativistic, second, the dynamic system is the principal point in the description of stochastic particles, and I do not believe that defects of the dynamic system can be corrected by means of its interpretation. At this stage of investigations I do not try to explain experimental data. At first, I should like to investigate properly the Dirac particle as a dynamic system. As concerns the connection between the dynamic disquantization and the one-particle interpretation, which the referee prescribes to my paper, I should like to declare, that such a connection is absent, in particular, because I do not use any interpretation at all except for that, which follows directly from dynamics.

Referee:

3) The author claims that the Dirac equation is not relativistic because it contains the gamma-matrix vector. As an example which should demonstrate the nonrelativistic character of the Dirac equation, the author considers Eq.(9.4) which is a covariant form of the nonrelativistic Eq.(9.3). The Eq.(9.4) contains vector L. I do not wish to discuss now mathematical proof of the relativistic character of Dirac equation (see the textbooks pointed out above), I just take an experimental-fitting (excuse me again) point of view: a system can be considered as relativistic if there are no ways to find a preferred inertial frame. From this point of view there is big difference between Eq. (9.4) and the Dirac equation. In the system (9.4) there is the preferred frame where $L = L_0 = (c, 0, 0, 0)$: in all frames the components of the vector L can in principle be measured and will be different in different frames. So, an observer can always say if she/he is moving (L not equal L_0) or at rest ($L = L_0$). This is not the case for the Dirac equation. There is no way to measure gamma-vector: all directions in gamma-vector space are equivalent.

Author's comment:

I agree with the referee that the analogue is not a proof. The example (9.1) - (9.4) is only an illustration of the fact, the relativistically covariant form of dynamic equations is not yet a proof of the relativistical character of the dynamic equations. (Most researchers are sure, that to prove relativistical character of description, it is sufficient to present dynamic equations in the relativistically covariant form. All proofs of relativistical character of the Dirac equation are founded on this belief). In reality the proof is founded on the theorem (J. L. Anderson, Principles of relativity physics. Academic Press, New-York, 1967, pp 75-88), which states, that the group of symmetry of dynamic equations, written in the relativistically covariant form, is determined by the group of symmetry of absolute objects. The absolute objects are those quantities, which are the same for all solutions. In the case of the Dirac equa-

tion the γ-matrices are the absolute objects, which determine the group of symmetry of the Dirac equation. (In reality it is of no importance, whether the absolute objects are vectors, or matrix vectors, only the transformation law (or the symmetry group) is of importance). At this point there is a lection. Some researchers consider that γ-matrices γ^k form a 4-vector, other researchers consider γ-matrices γ^k as four scalars, referring to the fact that γ-matrices are not changed at the Lorentz transformations. This problem is discussed in ref. [25] (physics/0412032) in details.

The real proof of nonrelativistic description of the internal freedom degrees of the Dirac particle is obtained after transformation of dynamic variables. After this transformation the action (4.1) is described in terms of tensor variables. At this transformation the γ–matrices are eliminated, the constant unit timelike 4-vector f^k (absolute object) appears instead of γ-matrices. The transformation is rather complicated and bulky. It can be found in ref. [25] (physics/0412032), where the action (4.1) is transformed to the form

$$\mathcal{A}_{\mathrm{D}}[j,\varphi,\kappa,\boldsymbol{\xi}]=\int\mathcal{L}d^4x,\qquad\mathcal{L}=\mathcal{L}_{\mathrm{cl}}+\mathcal{L}_{\mathrm{q1}}+\mathcal{L}_{\mathrm{q2}}\tag{1}$$

$$\mathcal{L}_{\mathrm{cl}}=-m\rho-\hbar j^i\partial_i\varphi+\hbar j^s\varepsilon_{iklm}\mu^i\partial_s\mu^k z^l f^m,\qquad\rho\equiv\sqrt{j^l j_l}\tag{2}$$

$$\mathcal{L}_{\mathrm{q1}}=2m\rho\sin^2(\frac{\kappa}{2})-\frac{\hbar}{2}S^l\partial_l\kappa,\tag{3}$$

$$S^k=\xi^k\rho+j^k\left(\xi^s f_s\right),\qquad k=0,1,2,3\tag{4}$$

$$\mathcal{L}_{\mathrm{q2}}=\hbar\rho\varepsilon_{iklm}q^i(\partial^k q^l)\nu^m\tag{5}$$

$$q^k=\frac{j^k+f^k\rho}{\sqrt{2\rho\left(j^s f_s+\rho\right)}},\qquad q^k q_k=1\tag{6}$$

$$\mu^i=\frac{\nu^i}{\sqrt{2(1-\nu^l z_l)}}.\tag{7}$$

$$\nu^k=\xi^k-f^k\left(\xi^s f_s\right),\qquad k=0,1,2,3;\qquad\nu^l\nu_l=-1\tag{8}$$

where φ is a scalar, κ is a pseudoscalar, the quantities j^k, q^k,f^k are 4-vectors and the quantities S^k, μ^k, ν^k, z^k are 4-pseudovectors. Here f^k is a constant unit timelike 4-vector, which is not fictitious. z^k is a constant unit spacelike 4-pseudovector, which appears to be fictitious. If one ignores the internal degrees of freedom (considering them as infinite rigid), the description of the Dirac particle becomes to be relativistic, because only internal degrees of freedom are described nonrelativistically. Note that the dynamic disquantization (which is disliked by the referee) has no relation to the problem of nonrelativistic character of the Dirac particle.

Finally, as one can see from the title and abstract, the paper is devoted to demonstration of effectiveness of dynamical investigation methods, which admit one to investigate those properties of the Dirac particle, which cannot be investigated by conventional quantum methods, restricted by the quantum principles. The paper does not pretend to explanation of any new experimental data. It is the next stage *of investigation, when the Lagrangian of the Dirac particle will be set free from its defects. I admit that the referee may have another approach to the problem, but it is cannot be a reason for rejection of the paper, because there are no mistakes in it.*

3 Comments to the second report

The referee:

I cannot change my opinion on the paper: the paper cannot be published. My arguments remain the same and I repeat them in what follows in another words.

1) The author claims that the "high frequency solutions" were unknown previously. It is not true (see my previous report).

Author's comment:

It is misunderstanding. I stated only, that in transition to nonrelativistic approximation of the Dirac equation one obtains the Pauli equation as a rule (i.e. the high frequency solution are lost (Dirac ref. [1]). Such a result is a corollary of incorrect transition to the nonrelativistic approximation, when one neglect the highest derivatives with small parameter before them. Such respectable researchers as P.A.M Dirac use incorrect method of transition to the nonrelativistic approximation and obtain the correct results. But obtaining of correct results by means of incorrect mathematical methods is ***a fitting****. Using incorrect methods, one cannot be sure in obtaining of the correct result, and a use of incorrect methods needs an independent test of the obtained result. This is the main defect of fitting. The goal of the paper is a discussion of* **dynamical methods**. *In this context it is of no importance, whether or not high frequency solutions were known previously. I do not state that the high frequency solutions were unknown previously. I do simply ignore this question. The free Dirac equation is a linear partial differential equation with constant coefficients. Contemporary mathematical methods admit one to obtain the general exact solution of such an equation. In particular, one can obtain the high frequency solutions. Such things are trivial. By no means I pretend to discovery of the high frequency solutions , as well as to discovery of methods of working with differential equations with small parameter before the highest derivative.*

The referee:

2) The author claims in the comment that she/he does not use any interpretation of the Dirac wave function at all. This is not true. The author considers the wave function as a c-number function. This is what is usually called a one-particle interpretation. It is well known that such approach leads to paradoxes. In particular, it predicts of the presence of oscillating terms in the Dirac current (so called "zitterbewegung"). The author "discovers" such terms and suggests that these terms "will disappear as a result of electromagnetic emanation". In this respect I would like to recommend to the author to study famous Klein's Paradox.

Author's comment:

I do not use any interpretation, which is something external with respect to the dynamical system. For instance, in the second section the Schrödinger particle is *considered as a dynamical system, described by the action (2.1), whereas the relation (2.6) carries out the interpretation of the wave function. On one hand, the relation (2.6) is something external with respect to the dynamic system (2.1) and it describes the meaning of the wave function. On the other hand, von Neumann has shown, that if the relation of the type (2.1) is valid for all functions* $F(\mathbf{x}, \mathbf{p})$*, it is equivalent to the principles of quantum mechanics. Thus, in the Schrödinger representation (2.1), (2.6) the dynamics and quantum principles (interpretation of the wave function) appear to be separated in the form of two different relations. It is shown in the second section that the Schrödinger particle can be described completely by the dynamics (2.1)* ***only****, and the quantum principles (2.6) appear to be needless. The current* j^k *and the energy-momentum tensor* T^{ik} *are attributes of the dynamic system (2.1). Interpretation of the Schrödinger particle is carried out on the basis of quantities* j^k *and* T^{ik} *(dynamical interpretation). When it seems that the relation (2.6) can give some additional information (the momentum distribution) with respect to the dynamical interpretation in terms of* j^k *and* T^{ik}*, it appears that in reality the momentum distribution is the distribution over the mean momenta, which may be obtained in terms of* j^k*. Thus,* ***all physical information*** *on the Schrödinger particle is concentrated in the dynamical system (2.1).*

One should expect, that in the case of the Dirac particle (4.1) all physical information is also concentrated in the dynamic system (4.1). Even if there are some additional physical information contained in some relation of the type (2.6), one should investigate the dynamic system (4.1) at first, because the additional information is to be taken in the form which is compatible with dynamics (for instance, the relation (2.6) is nonrelativistic, and it is incompatible with the Dirac equation, which is supposed to be relativistic).

Note, that the dynamics and the quantum principles are separated in the form of two different relations ***only in the Schrödinger picture****. In the Heisenberg picture the quantum principles are incorporated into dynamics (and dynamical equations) in the form of operators or matrices. In this case we cannot investigate dynamics separately from the quantum principles. If we use the wave function as an operator (not c-number), we introduce the quantum principles into dynamics. If the wave function in the action (4.1) is not c-number (an operator), the action (4.1) ceases to be the action for a dynamic system in the conventional sense of the concept of a dynamic system. In this case we* ***may not use principles of classical dynamics****, and investigate the obtained "dynamic system" as a classical dynamic system.*

The referee states that I consider the wave function as a c-number, and he/she is quite right. I am investigate the dynamic system by means of dynamical methods. It is the goal of my paper. If there are difficulties and paradoxes, it is not my fault, I am not responsible for correctness of the dynamic system, I am responsible **only for correctness of the investigation** *of this dynamic system***,** *because a manifestation of the consecutive application of the dynamical investigation methods is the goal of my paper. How to correct defects of the Dirac particle (states with negative energy, nonrelativistic character of internal degrees of freedom) is a special question, which is not considered in this paper. I believe, that one should modify* *the Dirac dynamic system, because it is the simplest way. (Additional constraints (quantum principles), imposed on dynamics must be compatible with dynamics, and a correction of additional constraints is more complicated). Besides, I believe that all physical information (and its defects) are concentrated in the dynamic system. The wave function is interpreted simply as a method of the dynamic system description. As to oscillating terms and connection of them with the zitterbewegung, I do not pretend to priority in obtaining of these results. I demonstrate simply the consecutive application of the dynamical methods of investigation.*

The referee:

Thus, I cannot consider the sections 5,6 of the paper as containing something new.

Author's comment:

The sections 5,6 contain a consecutive application of dynamical methods to the nonrelativistic approximation of the Dirac particle. In the process of transition to the nonrelativistic approximation the internal degrees of freedom, described by the variable ξ, appear. This variable describes some rotation. In the limit $c \to \infty$ the radius of rotation vanishes, although the rotation velocity does not vanish, in general, as it is shown in section 7. It means that the Dirac particle has internal degrees of freedom, which remain in the nonrelativistic approximation. I admit that this result has been obtained early in other form (for instance, as the zitterbewegung), but as far as know, internal degrees of freedom of the Dirac particle were not considered aforetime.

The referee:

I would like to stress only that in the paper the author does not point out that the obtained results are evidences of defects of the approach. If the author is really holding the point of view that the obtained results are defects of the approach (as it follows from the COMMENT) then she/he should stress it in the paper. But I stress again that I cannot consider these difficulties as new ones.

Author's comment:

I did not understand exactly, defects of whose approach are kept in mind (my approach, or the conventional one). If the referee keeps in mind, that I should formulate defects of the conventional approach, I agree with him, but it is rather difficult problem, because such a presentation needs a large volume. But the paper is rather volume even without this investigation. I cannot state in advance, whether it will be the known difficulties, or may be some of them will be new. I think, that this question of priority is of no importance in the given context, because the goal of the paper is a manifestation of the dynamical methods capacities.

The referee:

3) The Dirac lagrangian is Lorentz invariant if the Dirac wave function is considered as a spinor and gamma-matrices do not transform under the Lorentz transformations (another point of view, when gamma-matrices form a 4-vector is also possible, at least at this stage). This is a well established fact. The author makes change of variables in the invariant lagrangian and obtains the lagrangian presented in the non-relativistically invariant form. Then the author concludes that the Dirac

particle is not relativistic. This is a wrong logic. To be sure that any lagrangian is not relativistic, one has to know transformation rules of all objects entering the lagrangian. The author does not specify any transformation rules. But it is clear, for example, that ξ_α is not a spatial part of any 4-vector (it follows from Eq(6.12)). Therefore, the transformation rule for ξ is not trivial. Thus, to claim that the Dirac equation is not relativistic, the author should provide the readers with the transformation rules of the all dynamical variables. Note that these transformation rules must be consistent with a condition that r.h.s. of Eqs. (6.3) is a Lorentz spinor. It is clear from the last condition together with the fact that the Dirac lagrangian in terms of ψ is invariant, that the final result (Eq. (6.14)) must be Lorentz invariant.

Author's comment:

I understand the doubts of the referee on nonrelativistical character of the Dirac equation, because I myself was shocked, when I had obtained the unexpected result, transforming the Dirac equation to a description in hydrodynamic variables (*Advances in Applied Clifford Algebras*, **5**, pp 1-40, (1995)). *At first, I thought that it was a mathematical mistake, but further investigations have shown, that it is not so.*

At first, about a possibility of transformation of the relativistically invariant Lagrangian into the non-relativistically invariant one as a result of a change variables. A simple example (9.3) -(9.4) shows that it is possible, if the relativistically invariant Lagrangian contains absolute objects (in the given case γ-matrices), which are eliminated as a result of change of variables. In the case of example (9.3) -(9.4) the "relativistically covariant" equation (9.4) turns into non-relativistically covariant relation (9.3) after a change of variables, containing elimination of the absolute object (4-vector l_k). In the given case the change of variables is trivial. It consists in assignment of numerical values to the vector l_k. The Anderson's theorem, which has been cited in the paper, also demonstrates a possibility, that a formally relativistically covariant expression may be in reality nonrelativistic, if it contains absolute objects (4-vectors).

I should like to note that the strange relation (6.12) is a result of elimination of the γ-matrices. It does not depend on whether matrices γ are considered to be vectors, or scalars, because the current j^k is a 4-vector and $\rho = \sqrt{j_k j^k}$ is a 4-scalar in both cases. The denominator in the expression (6.12) contains the strange expression $\rho + j^0$, which is a sum of a scalar with a time component of a 4-vector. The relativistic Lagrangian is not to contain such expressions, but, maybe, the expression $\rho + j^0$ will be compensated after substitution of ξ^α into Lagrangian. To test, whether Lagrangian is invariant with respect to the Lorentz transformation, there is no necessity to consider transformation of the wave function ψ in the representation (6.3), as the referee suggested. Such a test is rather complicated and bulky. It is sufficient to eliminate the variables ξ^α from the Lagrangian, using the relation (6.12), which expresses ξ^α via tensor variables j^k, and S^k. But the simplest method of the test of the relativistic character of the Lagrangian is as follows. Instead of the three variables ξ^α, $\alpha = \{1, 2, 3\}$, defined by (6.12), one introduces four variables $\tilde{\xi}^k$,

$k=0,1,2,3$ *by means of the relation*

$$\tilde{\xi}^k=\rho^{-1}\left[S^k-\frac{j^k S^l f_l}{c(\rho+j^l f_l)}\right], \qquad k=0,1,2,3; \qquad \rho\equiv\sqrt{j^l j_l}, \tag{9}$$

where f_k is an unit constant timelike 4-vector. In the considered coordinate system, where $f_k=\{c,0,0,0\}$, the variables $\xi^\alpha=\tilde{\xi}^\alpha$, $\alpha=1,2,3$. The variable $\tilde{\xi}^k$ is a 4-pseudovector, as it follows from the presented expression. In this case there is no necessity to eliminate the variable ξ^α. It is sufficient to replace the variable ξ^α by the 4-pseudovector $\tilde{\xi}^k$. (Formally the presented relation is an expression of the non-relativistic relation (6.12) in the relativistically covariant form by means of introduction of a proper absolute object (4-vector f_k)). The obtained Lagrangian contains the unit constant 4-vector f_k. Being expressed via tensor variables $\tilde{\xi}^k$, f_k, ..., the Lagrangian has the relativistically covariant form, but it contains in general, the absolute object (the constant 4-vector f_k). If dependence of the Lagrangian on the 4-vector f_k is fictitious, the Lagrangian is relativistically invariant. If the Lagrangian depends really on f_k, the Lagrangian is not relativistically invariant. (See example (9.3) - (9.5) in the paper). In my comment to the first report of the referee the expression for Lagrangian of the Dirac particle contains two unit constant 4-vectors f_k and z_k. The 4-vector z_k appears to be fictitious, whereas the 4-vector f_k is not fictitious. Hence, the Lagrangian for the Dirac particle is not relativistically invariant. In expressions (1) – (8) all quantities are tensors, or pseudotensors (tilde in $\tilde{\xi}^k$ is omitted) , but the quantities f^k and z^k are absolute objects, which are introduced, to transform the quantities (6.12) into components of the 4-pseudovector. The same is valid for the constant 4-vector z^k, which appears to be fictitious.

Now, let me explain, why the Anderson's theorem is applicable in the case, when matrices γ^k form a 4-vector and ψ is a scalar, and why it is not applicable, when γ^k are scalars and ψ is a spinor. The fact is that, the Anderson's theorem is proved for the case of "natural" Lorentz transformation of coordinates, when the transformation law contains only transformed quantities. For any tensor T_l^{ik} it has the form

$$T'^{i'k'}_{l'}=\frac{\partial x'^{i'}}{\partial x^i}\frac{\partial x'^{k'}}{\partial x^k}\frac{\partial x^l}{\partial x'^{l'}}T_l^{ik}$$

For the quantities γ^k,ψ it has the form

$$\psi=\psi', \qquad \gamma'^{k'}=\frac{\partial x'^{k'}}{\partial x^k}\gamma^k$$

These transformations are natural in the sense, that they contain only coefficients $\frac{\partial x'^{k'}}{\partial x^k}$ of the Lorentz transformation and the transformed quantities themselves.

However, in addition the Dirac Lagrangian is invariant with respect to a group of unitary transformations U

$$\tilde{\psi}=U\psi, \qquad \tilde{\gamma}^k=U^{-1}\gamma^k U, \qquad U^{-1}=U^+$$

The unitary group is powerful enough, to transform $\gamma'^{k'}=\frac{\partial x'^{k'}}{\partial x^k}\gamma^k$ *into* γ^k *for any Lorentz transformation, described by coefficients* $\frac{\partial x'^{k'}}{\partial x^k}$*. Accompanying any natural Lorentz transformation by a proper unitary transformation* $U=U\left(\frac{\partial x'^{k'}}{\partial x^k},\gamma^k\right)$ *we obtain as a result*

$$\tilde{\psi}=U\left(\frac{\partial x'^{k'}}{\partial x^k},\gamma^k\right)\psi,\qquad \tilde{\gamma}^k=\gamma^k$$

where the transformed $\tilde{\psi}$ *depends on the* γ*-matrix representation. This transformation is known as the spinor transformation. It is not natural in the sense, that the transformation of* ψ *contains* γ*-matrices and depends on the* γ*-matrices representation. The Anderson's theorem is not proved for such combined transformations, and it is not applicable in this case. Note that this question, as well as other like questions, are discussed in paper (physics/0412032), which is devoted to discussion of the relativistic invariance of the Dirac particle.*

Essentially the nonrelativistic character of the internal degrees of freedom, described by the variables ξ^α*, starts from the strange relation (6.12). This relation appears after elimination of* γ*-matrices. Why has the expression (6.12) such a strange form, which contains the sum of a scalar with the time component of 4-vector? There is no exact answer for this question. Maybe, the reason lies in the fact that the matrix* γ^0 *plays a double role. On one hand,* γ^0 *is the time component of the matrix vector* γ^k*,* $k=0,1,2,3$*. On the other hand,* γ^0 *is a scalar in the relation* $\bar{\psi}=\psi^*\gamma^0$*.*

The referee:

Apart from that, I have a question concerning to parameterization (6.3). Is the expression (6.3) parameterizing all spinors? In particular, is it parameterizing solutions of the Dirac equations with both negative and positive energy?

Author's comment:

Eight independent variables: $A^2,\varphi,\ \kappa,\eta^\alpha,\alpha=1,2,3,\ n^\alpha,\alpha=1,2,3,\ n^2=1$ *in the relation (6.3) may be expressed via wave function* ψ *in the form*

$$A^2=j^kj_k,\qquad j^l=\bar{\psi}\gamma^l\psi$$

$$\cos\kappa=\frac{\bar{\psi}\psi}{j^kj_k}$$

$$\boldsymbol{\eta}=\frac{\mathbf{j}}{|\mathbf{j}|}\eta,\qquad \tanh\eta=\tanh|\boldsymbol{\eta}|=\frac{|\mathbf{j}|}{j^0}$$

$$\mathbf{n}=\frac{\boldsymbol{\xi}+\mathbf{z}}{\sqrt{2\left(\boldsymbol{\xi}\mathbf{z}+1\right)}},\qquad \xi^\alpha=\rho^{-1}\left[S^\alpha-\frac{j^\alpha S^0}{(j^0+\rho)}\right],\qquad \alpha=1,2,3;\qquad \rho\equiv\sqrt{j^lj_l}$$

$$S^l=i\bar{\psi}\gamma_5\gamma^l\psi,\qquad l=0,1,2,3$$

where z is an arbitrary constant 3-vector. Finally, the variable φ *is defined as the irrotational part of the 4-vector field*

$$\frac{i\left(\bar{\psi}\partial_k\psi-\partial_k\bar{\psi}\cdot\psi\right)}{2\sqrt{j^lj_l}}=\partial_k\varphi+\varepsilon_{klsm}\partial_lB^{sm}$$

where B^{sm} is some antisymmetric 4-pseudotensor field. As it follows from this relation, the variable φ satisfies the equation

$$\partial^k \partial_k \varphi = i\partial^k \frac{\left(\bar{\psi}\partial_k \psi - \partial_k \bar{\psi} \cdot \psi\right)}{2\sqrt{j^k j_k}}$$

which always has a solution. It means that for any wave function ψ there exists at least one set of such parameters $A, \varphi, \kappa, \boldsymbol{\eta}, \mathbf{n}$, that ψ may be represented in the form

$$\psi = Ae^{i\varphi + \frac{1}{2}\gamma_5 \kappa} \exp\left(-\frac{i}{2}\gamma_5 \boldsymbol{\sigma\eta}\right) \exp\left(\frac{i\pi}{2}\boldsymbol{\sigma}\mathbf{n}\right) \Pi$$

The referee:

4) I “dislike” the “dynamic disquantization” (DD) for two reasons. First, during the DD some degrees of freedom disappear, some do not. And it seems that there is no way to control how disappearing degrees suppressed with respect to the remaining one (at least the author does not provide the readers with any prescriptions). Second, DD uses a current associated with the global phase rotations of the wave function. The question is what one has to use to perform DD in systems where the current is absent (for example, real scalar field, majorano field, electromagnetic field), or in systems where there are several different currents associated with different symmetries? Another question is why one does not use a momentum which is, in contrast to the current, inherent in any system? Thus, it seems to me that DD as a general method is not consistent and one may obtain any “miraculous” results like internal structure of the Dirac particle, following this way.

My comment:

Dynamic disquantization is only a **method of investigation.** *Maybe, the title "dynamic disquantization" is not quite successful, because it associates with the quantum nature of the microparticles, with quantum principles and other conceptual things. In reality, it is simply a method of investigation of a continuous dynamic system, by means of their projecting onto dynamic systems with finite degrees of freedom. The result of projecting depends on the choice of the vector field j^k, which must be an* ***attribute*** *of the investigated continuous dynamic system. If we have several vector fields j^k, which are attributes of the dynamic system, we can construct several dynamic disquantizations. These several DDs may be different. Each DD describes some side of the investigated continuous dynamic system. The vector field j^k may be 4-current, 4-momentum, or one of eigenvectors of the energy-momentum tensor. All these vectors are attributes of the investigated dynamic system. In the case, when the dynamic systems has a vanishing current, one may use the timelike eigenvector of the energy-momentum tensor. Of course, in this case we may not state, that in applying DD we obtain a classical analog of the quantum system in the sense of quantum principles. The classical analog in the sense of the quantum principles may not exist at all. In general, the DD is a useful investigation procedure, if we do not look at it from the point of view of quantum principles. One should relate to dynamic disquantization in the same way, as we relate to the operation of* *the partial differentiation (sometimes it is useful, sometimes it is not). Sometimes DD is a procedure inverse with respect to the quantization procedure, sometimes it is not so. I do not see any problems here.*

Using of the DD is not an evidence of internal structure of the Dirac particle. It exists independently, and it manifests itself in the fact, that the Dirac equation has eight real components, whereas the Pauli equation contains only four. What do describe additional differential equations? Antiparticle? But for description of antiparticle it is sufficient to introduce only an additional dichotomic variable, for instance, $\mathrm{sign}p_0$ (p_0 *is the time component of 4-momentum). There is no necessity to introduce four additional differential equations. These four additional (with respect to the Pauli equation) differential equations are necessary for description of additional degrees of freedom. The DD is used only for investigation of the internal structure, but not for a proof of its existence.*

In general, the referee looks at my paper from the viewpoint of quantum principles and demands, that it explain quantum effects and contain new essential physical results. However, as it follows from the title, my paper is devoted to consideration of purely dynamical methods of investigation, which admit one to discover such sides of the quantum dynamic system which are hidden under the constraints, imposed by principles of quantum mechanics. It is another question, whether these hidden properties are useful, or they are defects. Consideration of these hidden properties and elimination of defects is the next stage of investigation, which lies outside the framework of the paper. Elimination of defects of the investigated dynamic system (Dirac particle) is possible, only if we have investigated it properly.

Application of dynamical methods of investigation to the Dirac system shows, that the Dirac particle has internal degrees of freedom, described nonrelativistically. Let us stress that it is a result of consecutive theoretical investigation. Any references to experimental data are irrelative. Only possible mistakes in my investigation may be important. The referee has failed to find defects or mistakes in my investigation, and I do not see reasons for rejection of the paper. The fact that the referee dislikes some introduced investigation procedures may not be a reason for rejection.

4 Concluding remarks

The paper [1] was rejected from publication in the journal on the basis of the referee's report. It is quite reasonable for the pioneer papers, which cannot be evaluated correctly by the average referees. The peer review system is not appropriate for evaluation of pioneer papers at the critical points of the physics development. We hope that publication of the comments to the referee's review can reduce in some measure defects of the peer review system.

References

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[5] Yu. A. Rylov, Is the Dirac particle completely relativistic? *physics/0412032*.

[6] Yu.A. Rylov, Dirac equation in terms of hydrodynamic variables, *Advances in Applied Clifford Algebras*, **5**, pp 1-40, (1995).