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Spectra of energized pick-up ions upstream of the two-dimensional heliospheric termination shock

II. Acceleration by Alfvénic and by large-scale solar wind turbulences

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Abstract. It is generally envisaged that pick-up ions originating by ionization processes from interstellar neutral atoms in the region of supersonic solar wind flow eventually act as seed population for anomalous cosmic rays. It is, however, fairly unclear till now where and how the necessary energization from KeV- to 10 MeV-energies takes place. Here we consider the continuous stochastic acceleration of pick-up ions at their convection to the outer heliospheric regions both by small-scale Alfvénic turbulence and by coherent nonlinear large-scale fluctuations of solar wind velocity and magnetic field. For these latter fluctuations we develope a new turbulence concept by which we describe the average effect of corotating interaction regions in energizing pick-up ions. It is shown that large-scale turbulence is responsible for the acceleration of pick-up ions from 10 to 100 KeV/nucleon while the preceded, primary pick-up ion acceleration from 1 to 10 KeV/nucleon is done by small-scale Alfvénic turbulence. Our results nicely can fit ULYSSES and VOYAGER data on energetic particles. We also confirm by our theory that a preferable acceleration of helium compared to hydrogen pick-ups occurs to 100 KeV/nuc energies. Also the reflection rates at the termination shock are favourable for helium pick-ups, and thus we expect injection rates into the ACR regime favorable for ACR helium. In no case within the frame of our theoretical studies, the typical "anomalous cosmic ray" energies of the order of 10 MeV/nucleon will be achieved before arrival of the pick-up ions at the shock. We can show, however, that the percentage of pick-up ions undergoing reflection at the shock will be much increased by the pre-acceleration operating upstream of the shock. In ongoing reflection processes of first and higher orders the ions permanently gain in energy till finally they are diffusively decoupled from the shock and can reappear in the inner solar system.

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1. Introduction to the pick-up ion scenario

The origin of anomalous cosmic rays (ACRs) in the heliosphere still now is an unsolved problem of heliospheric research. It is, however, accepted at present that interstellar neutrals penetrating into inner parts of the heliosphere become ionized there, and as ions are picked up immediately by the interplanetary magnetic field frozen into the solar wind. Some fraction of these particles is then accelerated to cosmic ray energies near or at the solar wind termination shock by means of diffusive first-order Fermi or by shock drift processes (see Axford et al., 1977; Pesses et al., 1981; Scholer, 1985; Potgieter & Moraal, 1988; Jokipii, 1992; Lee et al., 1996; Zank et al., 1996). As it follows from the argumentation of Jokipii (1992), only acceleration of particles at the quasi-perpendicular termination shock can explain the observed spectra of ACRs. However, in the framework of his model description there still exist some unresolved problems. Probably of prime importance amongst these problems is the question how pick-up ions are eventually injected into the ACR regime. After several detailed studies (see Chalov et al., 1995; Chalov & Fahr, 1996) we are convinced that the energization of pick-up ions essentially takes place as a two-step process: During their propagation to the outer parts of the heliosphere pick-up ions not only suffer pitch-angle scattering but in addition are systematically accelerated by Alfvénic turbulence, magnetosonic turbulence (or transit-time damping), and by interplanetary shock waves connected with corotating interaction regions (CIRs) and merged interaction regions (MIRs) (see Fisk et al., 1974; Fisk, 1976a/b/c; Klecker, 1977; Lee, 1983; Fisk, 1986; Isenberg, 1987; Bogdan et al., 1991; Petuhov & Nikolaev, 1993; Chalov et al., 1995; Fichtner et al., 1996). Thus upstream of the termination shock the actual energy distribution function of pick-up ions is not anymore the initial KeV-shell, but has already developed a high-energy tail, and upon arrival at the shock selected particles from this tail can then undergo a first reflection at the shock (Chalov & Fahr, 1996) and can then be subject to further acceleration by the first-order Fermi or shock drift processes near the shock, eventually up to ACR energies.

In the present paper we develop a new numerical, two-dimensional model to study in more quantitative terms the process of the pre-acceleration of pick-up ions in the heliosphere and their subsequent reflection at the termination shock. The model describes the local production of pick-up ions, their convection, adiabatic deceleration in the expanding solar wind, and their stochastic acceleration by interaction with all kinds of solar wind turbulences. Different from our earlier paper (Chalov et al. 1995, Paper I), and as an important enlargement of the theoretical basis, we here study the transport and acceleration of pick-up ions with inclusion of two new aspects:

A) We study the acceleration in a two-dimensional heliosphere with the symmetry axis oriented along the bulk velocity of the local interstellar medium (LISM). The shape of the termination shock and plasma velocities in the heliosphere are taken from numerical calculations carried out by Baranov & Malama (1993, 1995).

B) As an even more important improvement with respect to our previous model we here include particle acceleration processes connected with the action of coherent large-scale fluctuations in the magnitudes of the solar wind velocity and magnetic field. These large-scale fluctuations are associated with the propagation of CIRs and WIRs and long-period changes of the interplanetary medium. The corresponding periods are of the order of several days, and are treated here as large-scale waves to describe the averaged energization effect of CIRs and MIRs. As correlation length of this large-scale turbulence one can take the mean distance between consecutive interplanetary shock waves (i.e. of CIRs and MIRs). Since the kinetic energy of these large-scale fluctuations is comparable with the kinetic energy of the solar wind, they must be identified as a powerful energy reservoir for particle accelerations in the heliosphere.

To solve the governing Fokker-Planck equation (FP-equation) in two spatial dimensions we again make use here (as in Chalov et al., 1995: i.e. Paper I) of the mathematical equivalence of this type of a partial differential equation with a coupled system of stochastic ordinary differential equations which describe individual trajectories of particles in phase space. This method has already been used in the recent past for numerical solutions of different problems by MacKinnon & Craig (1991), Achterberg & Krülls (1992), Roberge et al. (1993), Krülls & Achterberg (1994), Chalov et al. (1995), and Fichtner et al. (1996).

2. Brief sketch of the theoretical approach

2.1. Application of the SDE method to the pickup ion transport equation

Interstellar neutral particles penetrating into the heliosphere can be ionized by means of solar EUV/UV radiation, charge exchange with solar wind protons, and collisions with solar wind electrons. The freshly created ions are picked up by the solar wind magnetic field and are considered as a suprathermal ion population comoving with the solar wind (Holzer & Leer 1973, Fahr, 1973; Mobius et al., 1985; Isenberg, 1986). Pick-up ions by effective pitch-angle scattering are rapidly isotropized in the solar wind rest frame. In addition, on larger time scales we consider the operation of stochastic acceleration in the turbulent Alfvénic and large-scale wave fields, and the adiabatic deceleration in the expanding solar wind on the energy spectra of pick-up hydrogen and helium.

The isotropic distribution function of pick-up ions, $f(t, \underline{\boldsymbol{r}}, v)$, can be found as the solution of the following form of a FP-equation:

$$\frac{\partial f}{\partial t} + \underline{\boldsymbol{U}} \frac{\partial f}{\partial \underline{\boldsymbol{r}}} = \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2 D \frac{\partial f}{\partial v}\right) + \frac{v}{3}\frac{\partial f}{\partial v}\,\mathrm{div}(\underline{\boldsymbol{U}}) = S(\underline{\boldsymbol{r}}, v)\,. \tag{1}$$

where $\underline{\boldsymbol{U}}(\underline{\boldsymbol{r}})$ is the solar wind velocity, v is the velocity of pick-up ions in the solar wind rest frame, $D(\underline{\boldsymbol{r}}, v)$ is the energy diffusion coefficient for pick-up ions, and $S(\underline{\boldsymbol{r}}, v)$ is the local production rate of freshly ionized particles. We shall use a spherical coordinate system (r, ϑ, ϕ) with the Sun as its origin and a polar axis pointed to the upwind direction of the LISM flow (i.e. $\vartheta = 0$ = antiparallel to the LISM wind velocity). Introduction of the differential density of particles in phase space $\{r, \vartheta, v\}$ with $N = 2\pi r^2 \sin\vartheta\ v^2 f$ helps transforming Eq. (1) into the following form:

$$\begin{aligned}\frac{\partial N}{\partial t} &= -\frac{\partial}{\partial r}(uN) - \frac{\partial}{\partial \vartheta}\left(\frac{w}{r}N\right) \\ &-\frac{\partial}{\partial v}\left[\left(\frac{\partial D}{\partial v} + \frac{2D}{v} - \frac{v}{3r^2}\frac{\partial u r^2}{\partial r} - \frac{v}{3r\sin\vartheta}\frac{\partial w \sin\vartheta}{\partial \vartheta}\right)N\right] \\ &+\frac{1}{2}\frac{\partial^2}{\partial v^2}(2DN) + 2\pi r^2 \sin\vartheta\, v^2 S(r, \vartheta)\,,\end{aligned} \tag{2}$$

where $u = u(r, \vartheta)$ and $w = w(r, \vartheta)$ are the radial and tangential velocities of the solar wind. The FP-equation in its above derived form clearly shows its identity with the following gerneral form of a partial differential equation of second order:

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial x_i}(\mathscr{A}_i f) + \frac{1}{2}\frac{\partial^2}{\partial x_i \partial x_j}\left[\left(\underline{B}\,\underline{B}^T\right)_{ij} f\right]\,, \tag{3}$$

where $\underline{A}(\underline{\boldsymbol{x}}, t)$ is an n-dimensional advection vector, $\underline{\mathrm{B}}(\underline{\boldsymbol{x}}$,t) is an n-by-n diffusion matrix, and summations are implied where repeated subscripts and superscripts ($i, j = 1, 2, \ldots, n$), respectively, appear. Here these quantities $\underline{A}$ and $\underline{B}$ are defined by:

$$A = \left(u, \frac{w}{r}, \frac{\partial D}{\partial v} + \frac{2D}{v} - \frac{v}{3r^2}\frac{\partial ur^2}{\partial r} - \frac{v}{3r\sin\vartheta}\frac{\partial w \sin\vartheta}{\partial\vartheta} \right) ,$$
$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{2D} \end{pmatrix} . \quad (4)$$

As noticed earlier (e.g. see Gardiner, 1990), Eq. (3) can be replaced by a set of Ito-stochastic differential equations (SDE) of the following form:

$$d\underline{\boldsymbol{x}} = \underline{A}(\underline{\boldsymbol{x}}, t)dt + \underline{B}(\underline{\boldsymbol{x}}, t)d\underline{\boldsymbol{W}}(t) , \quad (5)$$

where $\underline{\boldsymbol{W}}(t)$ is an n variable Wiener process, and $d\underline{\boldsymbol{W}}(t) = \underline{\boldsymbol{W}}(t + dt) - \underline{\boldsymbol{W}}(t)$ is a vector of statistically independent "Wiener"-differential increments. To integrate Eq. (5) numerically a Cauchy-Euler finite-difference method is applied to the following system:

$$dr = u\,dt , \quad (6a)$$

$$d\vartheta = (w/r)\,dt , \quad (6b)$$

$$dv = \left(\frac{\partial D}{\partial v} + \frac{2D}{v} - \frac{v}{3r^2}\frac{\partial ur^2}{\partial r} - \frac{v}{3r\sin\vartheta}\frac{\partial w\sin\vartheta}{\partial\vartheta} \right) dt + \sqrt{2D}dW_v . \quad (6c)$$

Introducing a finite time-interval Δt, we can rewrite Eqs. (6a) through (6c) in the discrete form:

$$r_{i+1} = r_i + u_i\Delta t , \quad (7a)$$

$$\vartheta_{i+1} = \vartheta_i + (w_i/r_i)\Delta t \quad (7b)$$

$$v_{i+1} = v_i + a_i\Delta t + b_i\Delta W_v(t) , \quad (7c)$$

where $r_i = r(t_i)$, $\vartheta_i = \vartheta(t_i)$, $v_i = v(t_i)$, $t_{i+1} = t_i + \Delta t$,

$$a_i = \left(\frac{\partial D}{\partial v} + \frac{2D}{v} - \frac{v}{3r^2}\frac{\partial ur^2}{\partial r} - \frac{v}{3r\sin\vartheta}\frac{\partial w\sin\vartheta}{\partial\vartheta} \right)_i ,$$
$$b_i = \sqrt{2D_i} . \quad (8)$$

The initial values, r_0, ϑ_0, and v_0, are determined in accordance with the source term S (see below).

2.2. Actual pickup-ion production rates

Neutral LISM atoms of different elements penetrate the interface region between the solar wind and the interstellar plasma and eventually appear in the inner heliosphere if not subject to ionization processes before. Here these neutrals are ionized and thereby generate a specific population of pick-up ions. The relevant, local heliospheric pick-up ion production rates for species A are calculated by the following formula:

$$q_A(\underline{\boldsymbol{r}}) = n_A(\underline{\boldsymbol{r}})[\nu_{ph,A}(\underline{\boldsymbol{r}}) + \nu_{ex,A}(\underline{\boldsymbol{r}})] , \quad (9)$$

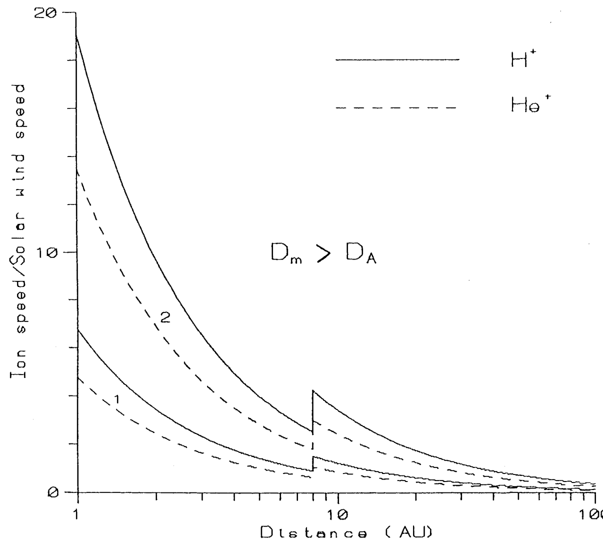

Fig. 1. Shown is the relative importance of small-scale Alfvénic (D_A) and large-scale magnetosonic turbulences (D_m) in the acceleration of the pick-up hydrogen (full curves) and helium (dashed curves) as expressed by diffusion coefficients: Each of the given curves divides the given plane $\{v/U; r/r_E\}$ in two regions. In the region above each curve $D_m > D_A$ is valid, while, in contrast, in the region below the curves $D_m < D_A$ holds. Case 1: χ = 0.1; case 2: χ = 0.4

where $n_a(\underline{\boldsymbol{r}})$ is the number density of the neutral species A, e.g. of neutral hydrogen and helium atoms in the heliosphere, and $\nu_{ph,a}(\underline{\boldsymbol{r}})$ and $\nu_{ex,a}(\underline{\boldsymbol{r}})$ are the local photoionization and charge exchange frequencies for species A, respectively. (For more details see e.g. Fahr & Rucinski, 1989; Rucinski et al., 1993; Fahr et al., 1995). In the present paper the densities $n_A(\underline{\boldsymbol{r}})$ are calculated within the selfconsistent twin-shock interface model developed by Baranov and Malama (1993,1995) and were determined with a method identical to that explained in Chalov et al.(1995). The pick-up ion production rate given by Eq. (9) then is related to the source in Eqs. (4) and (5) by

$$q(r, \vartheta) = \int v^2 S(r, \vartheta)\,dv . \quad (10)$$

Neglecting the velocity of the parent atoms, we may assume that the pick-up ion velocities at the event of their production are identical with the local solar wind velocity, i.e. $S \propto \delta(v-U)$. Fig. 1 of Paper I shows the pick-up ion production rates, q, per unit volume for H^+ and He^+ as functions of the distance from the Sun along the upwind axis.

3. Diffusion coefficients

In a first approximation interplanetary turbulences can be categorized as consisting of small-scale Alfvénic fluctuations and large-scale magnetosonic ones.

3.1. Alfvénic turbulence

To derive the energy diffusion coefficient in the case of Alfvénic turbulence the quasi-linear theory of resonant wave-particle interactions is used (Kennel & Engelmann, 1966; Lee, 1971; Jokipii, 1973; Schlickeiser 1989). The turbulent Alfvénic wave field with a power law spectrum is assumed to be composed of dispersionless Alfvénic waves. The mean-squared amplitude of the associated magnetic fluctuations can be written in the form

$$\langle \tilde{B}^2 \rangle = \sum_{j=\pm} \int P_j(k)\, dk\,, \tag{11}$$

where $P_j(k)$ is the power spectrum of the magnetic fluctuations. The sign of the wave number k determines the helicity and the subscript j denotes the direction of the wave propagation (i.e. up-field or down-field). It is assumed here that the wave field is unpolarized and has equal intensities in either of the two directions of propagation (symmetric or isotropic turbulence), i.e.

$$P_j(-k) = P_j(k)\,, \quad P_+(k) = P_-(k)\,. \tag{12}$$

Then we can write

$$P_j(k) = \frac{1}{2} A |k|^{-\gamma}\,, \tag{13}$$

where γ is the spectral index. From Eqs. (11), (13) we can find that

$$A = \frac{\gamma - 1}{2} \langle \tilde{B}^2 \rangle k_c^{\gamma-1}\,, \tag{14}$$

where $L_A = 2\pi/k_c$ is the correlation length of the turbulent Alfvénic wave field.

Under the above mentioned assumptions the energy diffusion coefficient can be written in the following form (see e.g. Isenberg, 1987)

$$D_A = \frac{\pi v_A^2}{\gamma(\gamma+2)} \left(\frac{q}{mc}\right)^{2-\gamma} B^{-\gamma} A\, v^{\gamma-1}\,, \tag{15}$$

where v_A is the Alfvénic velocity, q and m are the charge and mass of the ions, and B is the magnitude of the mean magnetic field. Furthermore we shall assume that

$$B = B_E \frac{r_E}{r}\,, \quad \langle \tilde{B}^2 \rangle = \langle \tilde{B}_E^2 \rangle \left(\frac{r_E}{r}\right)^{\alpha}\,, \quad L_A = L_{AE} \left(\frac{r}{r_E}\right)^{\delta}\,. \tag{16}$$

Then Eq. (15) can be written as

$$D_A = D_{A0} \frac{U_E^3}{r_E} \left(\frac{v}{U_E}\right)^{\gamma-1} \left(\frac{r}{r_E}\right)^{\gamma-\alpha+\delta(1-\gamma)}\,, \tag{17}$$

where

$$D_{A0} = \frac{\pi^2(\gamma-1)}{\gamma(\gamma+2)} \left(\frac{v_A}{U_E}\right)^2 \left(\frac{qBL_A}{2\pi mcU}\right)_E^{2-\gamma} \frac{r_E}{L_{AE}} \frac{\langle \tilde{B}_E^2 \rangle}{B_E^2}\,. \tag{18}$$

In Paper I we assumed that the correlation length L_A of Alfvénic turbulence has a constant value throughout the heliosphere. It, however, seems to us to be more reasonable to assume that L_A increases with solar distance because of the expansion of the wave structures in azimuthal direction at their convection by the solar wind to larger distances. From the theoretical investigation of the Alfvén wave transport in the expanding solar wind Jokipii (1973) has found that the length of Alfvén waves propagating along the convected magnetic field lines should increase according to the form given in Eq. (16) with a power index of $\delta = 1$. What concerns the spatial dependence of $\tilde{B}$ (also see Eq. (16)) we assume here that $\alpha = 3$. This dependence follows from the WKB theory of dissipationless transport of Alfvén waves in the solar wind without sources of spatially distributed wave power (Hollweg, 1974) and close to observations on the HELIOS spacecraft (Tu & Marsch, 1995). For the correlation length of the Alfvénic fluctuations of the magnetic field at the Earth's orbit we choose here $L_{AE} = 1.3 \cdot 10^{11}$ cm. This value is very close to one given by Jokipii & Coleman (1968). It is also convenient for the following considerations to introduce a dimensionless parameter which characterizes the intensity of Alfvénic fluctuations:

$$\chi = \langle \tilde{B}_E^2 \rangle / B_E^2\,.$$

3.2. Large-scale turbulence

Besides Alfvénic turbulence with correlation lengths of order of 0.01 AU spaceprobes also use to observe large-scale oscillations in the magnitudes of the solar wind velocity and magnetic field with spatial scales of several AU. As a rule these oscillations are connected with CIRs and MIRs, and contain the structures of large-scale interplanetary shock waves. Observations show that within these large scale waves $\delta U \simeq U$, where δU is the magnitude of large-scale oscillation of the solar wind velocity (see e.g Bialk and Dröge, 1993; Keppler et al., 1995). Thus the kinetic energy contained in the large-scale oscillations can be comparable with the kinetic energy of the quiet solar wind and energetic particles thus can effectively gain energy from these oscillations if resonantly scattered. It has been shown by Toptygin (1983) that the acceleration of particles by large-scale supersonic turbulence including shock waves is equivalent to the second-order Fermi process because, in a first order view, the acceleration at the shock fronts is compensated by the deceleration in the following rarefaction waves.

The theory we shall use here is valid when $\Lambda \ll L_m$, where Λ is the mean free path of pick-up ions with respect to scatterings by short-wavelength Alfvénic fluctuations, and when L_m is the correlation length of large-scale fluctuations of the solar wind. We may suppose here that the following relation is valid:

$$\tau_{\rm dif}/\tau_{\rm conv} = 3UL_m/v\Lambda \gg 1\,, \tag{19}$$

where $\tau_{\rm div} \cong L_m^2/K$ is the time of the diffusive propagation of particles over the distance L_m, $K = v\Lambda/3$ is the coefficient of the spatial diffusion corresponding to short-wavelength Alfvénic turbulence, and $\tau_{\rm conv} \cong L_m/U$ is the convection time for the passage of a wave with scale L_m. Condition (19) will be fulfilled in good approximation for lower energy particles.

Whenever one can assume this condition to be fulfilled, one can also adopt as well that the spatial diffusion is unimportant. Then the corresponding velocity diffusion coefficient $D_m(v)$ due to large-scale turbulence can be written in the following form (see Toptygin, 1983):

$$D_m(v) = \frac{v^2}{9} \int_0^\infty \langle \nabla \cdot \delta \underline{\boldsymbol{U}}(\underline{\boldsymbol{r}}, t) \; \nabla \cdot \delta \underline{\boldsymbol{U}}(\underline{\boldsymbol{r}}, t - \tau) \rangle d\tau \,, \qquad (20)$$

where the expression in brackets $\langle \ldots \rangle$ stands for a large-scale correlation average. Evaluating the correlation integral in Eq. (20) by the order of magnitude one can obtain

$$D_m = (\langle \delta U^2 \rangle v^2 / 9 L_m^2) \tau_c \,, \qquad (21)$$

where τ_c is the maximum correlation time. We can assume that this time is approximately given by

$$\tau_c \simeq L_m / \langle \delta U^2 \rangle^{1/2} \,. \qquad (22)$$

Then

$$D_m = \frac{\langle \delta U^2 \rangle^{1/2} v^2}{9 L_m} \,. \qquad (23)$$

The amplitude of interplanetary large-scale shock waves in CIRs and MIRs according to theoretical considerations decreases with the increase of the heliocentric distance (Whang & Burlaga, 1988). Thus we may assume here the following distance-dependence

$$\left(\frac{\langle \delta U^2 \rangle}{\langle \delta U^2 \rangle_E} \right)^{1/2} = \left(\frac{r_E}{r} \right)^\beta \,. \qquad (24)$$

Now by use of Eqs. (23), (24) the diffusion coefficient can be written in the following convenient form

$$D_m = D_{mE} \frac{U_E^3}{r_E} \left(\frac{v}{U_E} \right)^2 \left(\frac{r_E}{r} \right)^\beta \,, \qquad (25)$$

where

$$D_{mE} = \frac{\langle \delta U^2 \rangle_E^{1/2} / U_E}{9 L_m / r_E} \,. \qquad (26)$$

From the numerical MHD simulation of the shock wave propagation in the outer heliosphere (Whang & Burlaga, 1988) we then can deduce that $\beta \approx 0.7$. With regard to the correlation length L_m of large-scale turbulence it seems to be quite reasonable to assume that L_m is the mean distance between large-scale interplanetary shock waves. From numerous observations on PIONEER and VOYAGER spacecraft and numerical simulations of evolutions of recurrent solar wind structures in the distant heliosphere (see e.g. Whang & Burlaga 1988; Whang & Burlaga 1990a,b; Lazarus et al.1995) one can obtain that $L_m \approx 3$ AU. Besides that, it appears that a period-doubling of the large-scale fluctuations occurs between 5 AU and 10 AU (Burlaga, 1988; Whang & Burlaga 1990a). So we shall assume in our calculations that $L_m = 1.5$ AU at $r < 8$ AU and $L_m = 3$ AU at $r > 8$ AU. Observations of interplanetary shock waves connected with CIRs close to the Earth's orbit show that $\langle \delta U^2 \rangle^{1/2} \simeq U$. Thus if not mentioned in particular we shall assume in the following that $\langle \delta U_E^2 \rangle^{1/2} \simeq U_E$.

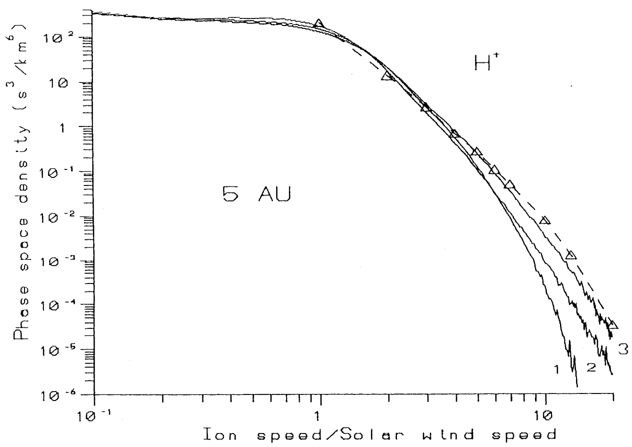

Fig. 2. The phase space densities of the accelerated pick-up hydrogen at $r = 5$ AU and $\vartheta = 110°$ for different values of χ: **1**: $\chi = 0.5$ and $\delta U = 0$; **2**: $\chi = 0.3$; **3**: $\chi = 0.5$. The triangles represent the ULYSSES observations of interplanetary pick-up hydrogen at 4.5 AU (Gloeckler et al., 1994) divided by a factor of three

Finally then the combined diffusion coefficient from Eq. (4) can be written in the form:

$$D(r, v) = D_A(r, v) + D_m(r, v) \,, \qquad (27)$$

where D_A and D_m are given by Eqs. (17) and (25). In Fig. 1 each curve divides the plane $(r/r_E, v/U_E)$ into two parts. In the region above a curve $D_m > D_A$, in the region below this curve $D_m < D_A$. One can see from Fig. 1 that the acceleration by large-scale turbulence is more important in the outer parts of the heliosphere and for high-velocity particles.

Recent ULYSSES data show that the magnitude of large-scale oscillations decreases with latitude (see Keppler et al., 1995). The intensity of forward and reverse shocks connected with CIRs sharply decreases at latitudes about $\pm 40°$. Only a very weak reverse shock has been observed up to now at 58° (see e.g. Gosling et al.1993, Phillips et al., 1995). Since we consider here latitude-independent diffusion coefficients, the results of our present calculations can thus only be applied to regions within $\mp 35°$ to the ecliptic plane, but neverthless some qualitative conclusions about acceleration of pick-up ions at higher latitudes can also be drawn.

4. Numerical scheme and results

We shall solve Eqs. (8a–c) in the region inside of the termination shock $r < r_{\rm sh}(\vartheta)$ where $r_{\rm sh}(\vartheta)$ is taken from model calculations by Baranov & Malama (1993, 1995). This region is divided into 20 radial layers and 36 sectors with respect to ϑ. If l and m are counting the numbers of the layer and sector, then $(r^{(l)}, \vartheta^{(m)})$ will be taken as the coordinate of the central point within the corresponding cell. Within each cell the pick-up ion production rate is taken to have one specific, constant value. Then the algorithm for the computation of energy spectra of pick-up ions upstream of the shock runs as follows: First we fix the numbers of a cell, l and m. After that we solve Eqs. (8a–c) with the initial conditions

$$r_0 = r^{(l)}, \quad \vartheta_0 = \vartheta^{(m)}, \quad v_0 = U(r_0, \vartheta_0)\,. \tag{28}$$

Then we calculate 10^4 trajectories starting at a point (r_0, ϑ_0, v_0). Each trajectory ends at the termination shock with $r = r_{\rm sh}$ and with a specific value of the velocity which on this trajectory is achieved just there. Then the represented particle is collected into a specific cell of the velocity space histogram which in our case has a cell spacing of $(\Delta v/U) = 0.1$. The resulting differential number density of the particles must of course then be based on the real pick-up ion production, i.e divided by 10^4 and multiplied by a number in accordance with the actual, integrated pick-up ion production rate within each cell under consideration (see Eqs. (9), (10)). This procedure is repeated for other values of l and m ($l = 1, \ldots, 20$; $m = 1, \ldots, 36$). Then all the number densities are summarized.

To evaluate the diffusion coefficient D_{0A} (see Eq. (18)) we take $\gamma = 5/3$, $v_A = 50\,{\rm km/s}$, $U_E = 450\,{\rm km/s}$, $B_E = 5 \cdot 10^{-5}$ G.

Fig. 2 shows the phase space densities of the accelerated pick-up hydrogen at $r = 5$ AU and $\vartheta = 110°$ for different values of χ. The curve 1 corresponds to the case when large-scale turbulence is ignored, i.e. $\delta U = 0$. The triangles are the ULYSSES data (Gloeckler et al.1994; Mall et al. 1996) divided by a factor of three. The interstellar pick-up hydrogen has been observed on ULYSSES during the passage of a CIR at 4.5 AU in October 1991. The number density of the observed pick-up hydrogen at this distance was about $9 \cdot 10^{-4}\,{\rm cm}^{-3}$. The corresponding theoretical density resulting from our present calculations is $3 \cdot 10^{-4}\,{\rm cm}^{-3}$. This deviation is connected with the fact that ULYSSES at that time had passed through the CIR with the enhanced densities of the solar wind plasma and pick-up ions, while in our theoretical model here we are describing average phenomena and thus can only consider the averaged values of the densities, velocities etc. One can see from Fig. 2 that the observed spectrum can be explained by the stochastic acceleration of pick-up ions simulaneously by both high-level Alfvénic turbulence ($\chi = 0.5$) and large-scale turbulence.

Figs. 3a,b show the energy spectra of the accelerated pick-up hydrogen in the upwind direction at 15 AU and 42 AU for different values of χ. The triangles are the VOYAGER-2 data taken from Gold et al. (1988) and Decker et al. (1995). VOYAGER-2 has observed accelerated ions ($Z \geq 1$) close to the ecliptic plane. One can conclude from Figs. 3a,b that it seems to be more natural to consider the observed ions as accelerated pick-up ions rather than pure solar wind particles or ACRs. Fig. 4 shows the radial distributions of the pick-up hydrogen with the kinetic energy about 64 keV in the upwind direction for different values of χ. The spatial gradients of the accelerated particles are very close to those measured by VOYAGER-2 up to distances of 43 AU (Decker et al., 1995) if $\chi > 0.1$. The curve 4 shows that the rate of the stochastic acceleration is very low at large distances ($r > 10$ AU) from the Sun (even at high level of Alfvénic turbulence at the Earth's orbit) as long as the acceleration by large-scale turbulence is not taken into account. It is interesting to note that our calculations predict considerable decrease of the radial gradients of the accelerated pick-up hydrogen at heliocentric distances $r > 50$–70 AU dependent on the magnitude of χ. The detection of this decrease will be a strong argument in favour of the interstellar origin of accelerated hydrogen in the outer heliosphere.

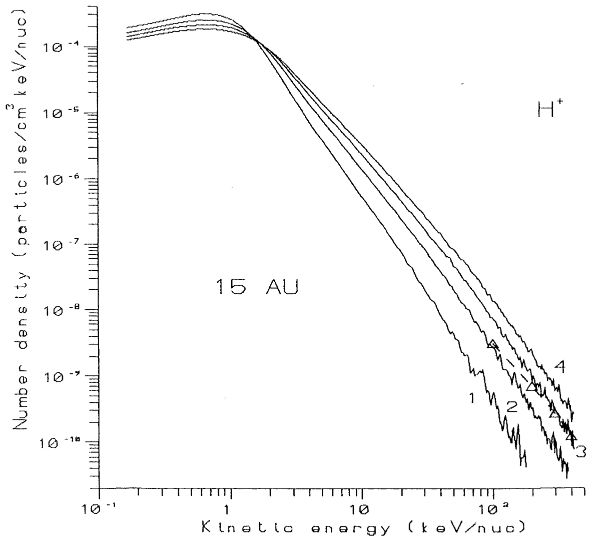

Fig. 3a. The energy spectra of the accelerated pick-up hydrogen in the upwind direction at 15 AU for different values of χ: **1**: $\chi = 0.1$; **2**: $\chi = 0.2$; **3**: $\chi = 0.3$; **4**: $\chi = 0.4$. The triangles are the VOYAGER 2 data (Gold et al., 1988)

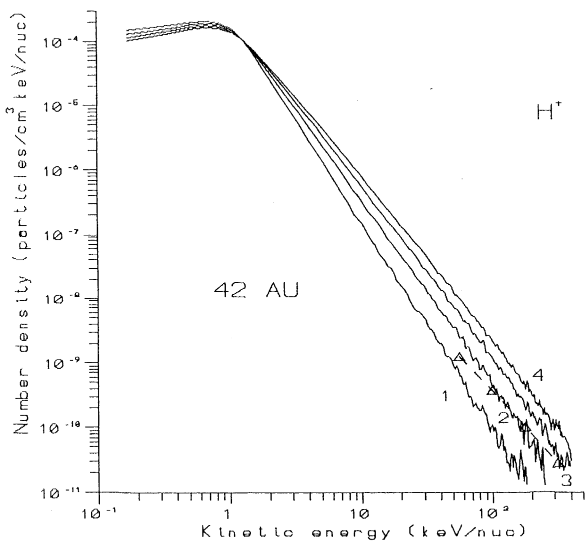

Fig. 3b. The energy spectra of the accelerated pick-up hydrogen in the upwind direction at 42 AU for different values of χ: **1**: $\chi = 0.1$; **2**: $\chi = 0.2$; **3**: $\chi = 0.3$; **4**: $\chi = 0.4$. The triangles are the VOYAGER 2 data (Decker et al., 1995)

Fig. 5 shows the spatial evolution of the energy spectra of the pick-up hydrogen in the upwind direction at $\chi = 0.2$. One can

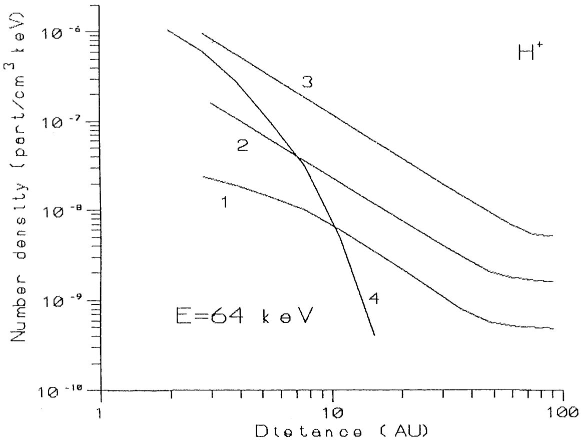

Fig. 4. The number densities of the pick-up hydrogen with kinetic energies of about 64 keV as functions of the radial distance from the Sun in the upwind direction for different values of χ: **1**: χ = 0.1; **2**: χ = 0.2; **3**: χ = 0.4; **4**: χ = 0.4, and δU = 0

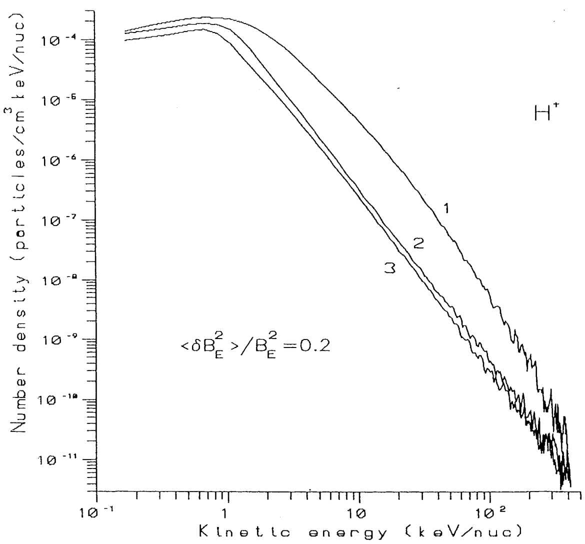

Fig. 5. The energy spectra of the accelerated pick-up hydrogen in the upwind direction at χ = 0.2 for different radial distances: **1**: 5 AU; **2**: 42 AU; 3: 91 AU

see that the shapes of the spectra are identical at large distances. Fig. 6 shows the energy spectra of the accelerated pick-up hydrogen and helium upstream of the frontal part of the termination shock ($r_{\rm sh}$ = 91 AU) for different values of χ. The helium spectra, for the sake of better comparison, are multiplied by a factor of 145. It is clear from Fig. 6 that large-scale fluctuations play the dominant role in the acceleration of pick-up ions in the outer parts of the heliosphere. The higher acceleration rate of the pick-up helium in comparison with that of pick-up hydrogen is a very eminent and interesting peculiarity of the results presented in Fig. 6. At first glance this result is rather unusual because the Alfvénic diffusion coefficient D_A decreases when the ion mass increases (see Eq. (18)), while the diffusion coefficient D_m is independent on the ion mass. We may, however, propose the following explanation of this astonishing result: Charge exchange reactions are the most important processes to ionize the neutral LISM hydrogen in the heliosphere. This reaction operates all over in the heliosphere, while the neutral helium mainly is ionized by the solar EUV radiation fairly close to the Sun ($r \ll 0.5$ AU). If we consider a small plasma volume near the termination shock then a relatively large part of pick-up helium in this volume (large in comparison with a corresponding part for the hydrogen) has been ionized already in regions close to the Sun. These pick-up ions thus have traveled from the innermost regions to the outer heliosphere undergoing accelerations for a long time. Hence they have been subject to acceleration processes for much longer times than those pick-up ions, like hydrogen pick-ups, of more local origin.

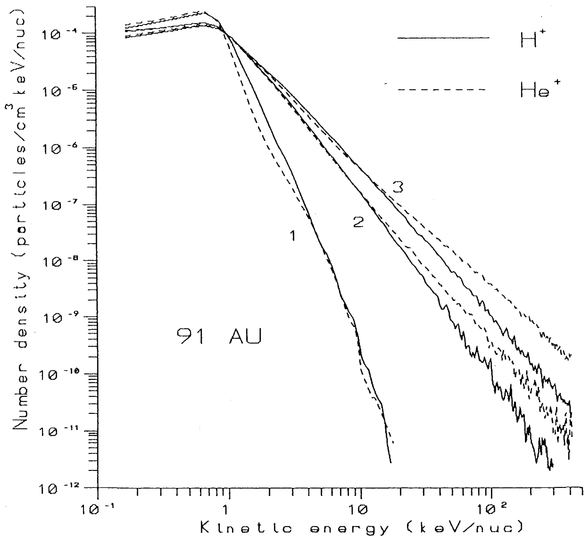

Fig. 6. The energy spectra of the accelerated pick-up hydrogen and helium in front of the nose part of the termination shock for different values of χ: **1**: χ = 0.4, and δU = 0; **2**: χ = 0.1; **3**: χ = 0.4. The helium spectra are multiplied by a factor of 145

5. Modelling of the pick-up ion reflection at the termination shock

When pick-up ions preaccelerated by solar wind turbulences to energies of several hundred keV eventually reach the termination shock, some fraction out of these ions then experiences further acceleration to 10–100 MeV energies by means of shock drift and diffusive acceleration processes (see e.g. Jokipii, 1992) and eventually at these energies can diffuse backwards to the inner part of the heliosphere as ACRs. Though the main principles of that procedure are agreed upon, it is nevertheless an unsolved problem, what fraction of the pick-up ions is converted into the ACRs (injection problem). In many recent papers the injection of pick-up ions into the ACR component is considered in terms of reflection of such ions at the termination shock (Liewer et al.1993, Giacalone et al.1994, Liewer et al. 1995, Ellison et al., 1995; Kucharek & Scholer, 1995; Zank et al., 1996; Lee et al., 1996; Chalov & Fahr, 1996). In the course of the reflection the kinetic energy of particles increases, and thus these particles can be considered as a seed population for ACRs. But, as a standard in these approaches, pick-up ions are taken to be distributed in velocity space of the solar wind frame by a shell-like, isotropic distribution function with a radius equal to the solar wind velocity (initial pick-up shell). In this case, only very few particles with relatively small velocities relative to the shock can be reflected by the electrostatic cross-shock potential wall. In Chalov & Fahr (1996) we have already emphasized the role of the pre-acceleration of pick-up ions by solar wind turbulence in determining their reflection probabilities at the termination shock. Here we continue this study, however, taking into account in addition the relevant acceleration processes by large-scale turbulence.

Since in the solar wind rest frame the gyroradius of a typical pick-up ion is much larger than that of a regular solar wind proton, and since the thickness of the shock front can be expected to be of the order of a few gyroradii of solar wind protons, one therefore, for the passage of preaccelerated pick-up ions, can approximate the shock by an MHD-discontinuity, neglecting the magnetic field overshoot and cross-shock potential that might appear in the transition region of simulated collisionless shocks (e.g. see Liewer et al., 1993, 1995; or Kucharek and Scholer, 1995). One single encounter of a pick-up ion with the shock involves many consecutive crossings of the ion Larmor orbit through the shock surface.

The reflection process thus is due to abrupt changes in both the strength and the direction of the magnetic field through the shock and operates for high velocity particles different from the reflection by the electric cross-shock potential ramp (see e.g. Decker, 1988). During the reflection the mean kinetic energy of pick-ions increases by approximately a factor of 10 due to the scatter-free shock drift acceleration process at the termination shock which in our case has an effective Mach number of about 3 (pick-up ion pressure included; see Figs. 10a–c of this paper). At these energies the spatial diffusion of particles still is only of marginal importance, but repeated reflections with consecutive energy gains can occur till spatial diffusion eventually becomes an important process. So one can consider the reflected pick-up ions of the first generation as a seed population for the first-order Fermi process.

Apparently this representation of the termination shock by a discontinuity is valid only at those heliolatitudes where the shock is quasi-perpendicular. The reflection efficiency of pick-up ions at the termination shock strongly depends on the angle between the shock normal and magnetic field as already demonstrated by numerical simulations carried out by Scholer & Terasawa (1990), Scholer (1993), Liewer et al. (1993), Kucharek & Scholer (1995), Chalov & Fahr (1996). It also strongly depends on the velocity distribution of pick-up ions arriving at the shock, as becomes evident from Chalov & Fahr (1996).

Recent observations on ULYSSES (Balogh et al., 1995; Forsyth, 1995) have confirmed that at least for low and moderate latitudes the interplanetary magnetic field is fairly close to the Parker spiral field and, thus, is almost azimuthal at large distances from the Sun. If the termination shock had a spherically symmetric surface then the lines of the solar wind magnetic field

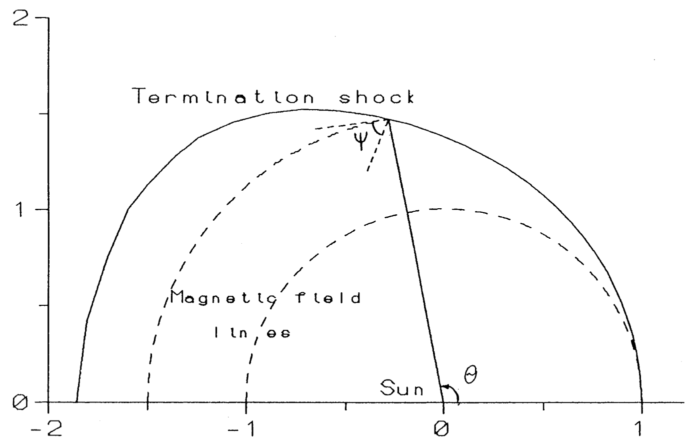

Fig. 7. The shape of the solar wind termination shock in the ecliptic plane as it follows from the twin-shock model by Baranov and Malama (1993)

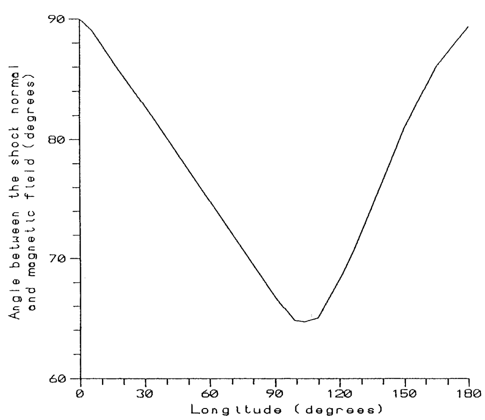

Fig. 8. The angle between the termination shock normal and the upstream magnetic field lines as a function of longitude

would be nearly perpendicular to the shock normal for the largest part of the shock surface. In reality the termination shock most likely has a nonspherical and prolate shape and field-directions within the ranges of the neutral sheath change periodically. The shock surface may most likely have elongations over the poles (see Fichtner et al. 1995 a,b), at least during solar minimum conditions (see Manoharan, 1993) and shall in addition have a pronounced upwind-downwind asymmetry (e.g. see Baranov et al., 1979; Matsuda et al., 1989; Baranov & Malama, 1993, 1995; Pauls et al., 1995). In the axially symmetric model of Baranov and Malama (1993) the angle, ψ, between the shock normal vector and the upstream magnetic field given by Parker(1963) varies with variation of the off-axis angle, ϑ (angle between the solar radian and the upstream interstellar wind axis, see Fig. 7). Fig. 8 shows the angle ψ as a function of longitude, ϑ, in a plane close to the ecliptic. Because the reflection rate of energetic particles at quasi-perpendicular shocks decreases when the shock normal angle, ψ, increases (see e.g. Decker, 1988), then the reflection efficiency and, consequently, the injection rate of pick-up ions

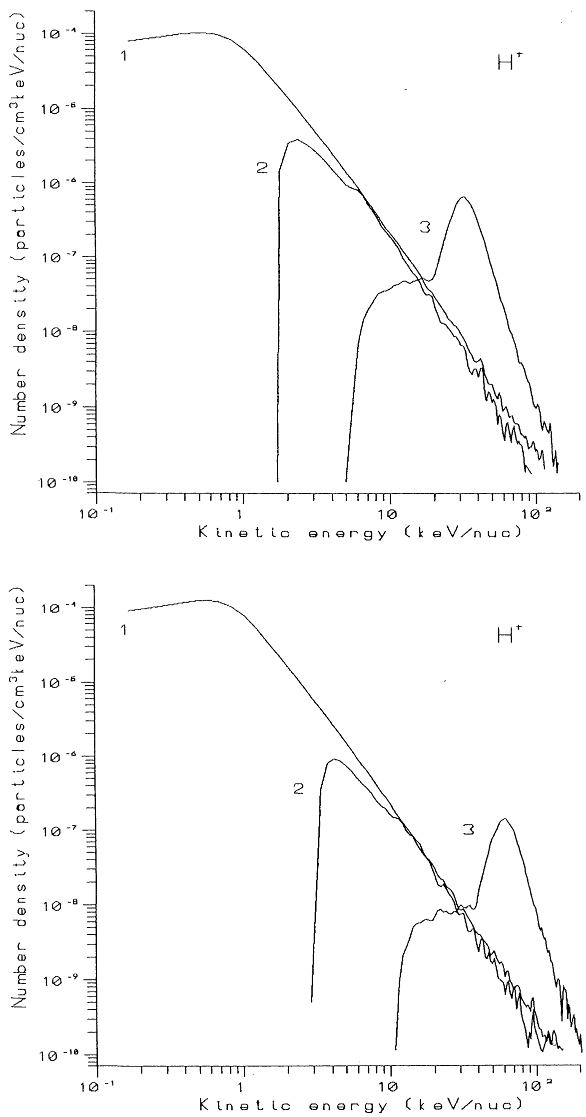

Fig. 9. The number densities of the incident pick-up hydrogen in front of the termination shock (curves 1), the number densities of incident particles for which the reflection conditions are fulfilled (curves 2), the number densities of reflected particles (curves 3). χ = 0.2. a) ψ = 70°, b) ψ = 75°, c) ψ = 80°

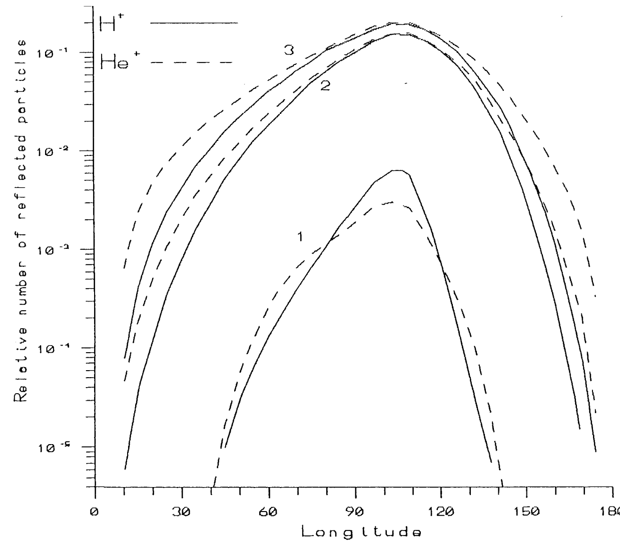

Fig. 10. Relative numbers, η, of reflected pick-up hydrogen and helium as functions of the longitude: **1**: χ = 0.4, and δU = 0; **2**: χ = 0.1; **3**: χ = 0.4

into ACRs will depend on longitude or the off-axis position. As it has been shown by Chalov (1993) this dependence can result in longitudinal gradients of low-energy ACRs near the termination shock.

In the present study as we have mentioned above we shall approximate the termination shock by a discontinuity. In this case the reflection conditions at the shock front can be derived from the conservation of the energy of a particle in the de Hoffmann-Teller frame (frame comoving with the merging point of the upstream solar wind magnetic field) and from the assumption that the magnetic moment of this particle is the same before and after the encounter with the shock front, i.e. is an adiabatic invariant. Strictly speaking the last assumption is true for nearly perpendicular shocks (see Toptygin, 1980) but numerical investigations have shown that this assumption is fairly good also for quasi-perpendicular cases (Terasawa, 1979). The pitch angle cosines in the upstream plasma frame corresponding to the cosines of the loss cone angles in the HT-frame can be written in the form (see Decker, 1988):

$$\mu_{\pm} = \varepsilon_1 b^{-1} \pm \left[(1 - b^{-1})(1 - b^{-1}\varepsilon_1^2)\right]^{1/2} , \quad (29)$$

where

$$\varepsilon_1 = (U_1/v)\sec\psi_1 \cos\delta_1 , \quad (30)$$
$$b = B_2/B_1 . \quad (31)$$

Here ψ_1 and δ_1 are the angles between the shock normal vector $\underline{\boldsymbol{n}}$ and the magnetic field vector $\underline{\boldsymbol{B}}_1$, and between $\underline{\boldsymbol{n}}$ and the solar wind velocity vector $\underline{\boldsymbol{U}}_1$, respectively. The quantity v is the velocity of pick-up ions in the solar wind frame. We shall assume here that $\psi_1 + \delta_1 = \pi/2$, i.e. magnetic field lines are perpendicular to the solar wind velocity.

Following Decker (1988) we can then conclude that:

1) a particle with the pitch angle cosine μ in the upstream solar wind frame is reflected at the shock if

$$-\varepsilon_1 < \mu \le \mu_+ \quad (\varepsilon < 1) , \quad (32a)$$

$$\mu_- < \mu \le \mu_+ \quad (1 \le \varepsilon_1 < b^{1/2}) . \quad (32b)$$

The kinetic energy of the particle after reflection is

$$E_R = E_I \left[1 + 2\gamma(\gamma - 1)^{-1}\beta^2\varepsilon_1(\varepsilon_1 + \mu)\right] , \quad (33)$$

where E_I is the initial kinetic energy, $\beta = v/c$, $\gamma = (1-\beta^2)^{-1/2}$;
2) a particle is transmitted downstream if

$$\mu_+ < \mu \leq 1 \qquad (\varepsilon_1 < 1), \qquad (34a)$$
$$-1 \leq \mu \leq \mu_- \text{ and } \mu_+ < \mu \leq 1 \qquad (1 \leq \varepsilon_1 < b^{1/2}), \qquad (34b)$$
$$-1 \leq \mu \leq 1 \qquad (\varepsilon_1 \geq b^{1/2}); \qquad (34c)$$

3) a particle does not interact with the shock if

$$-1 \leq \mu \leq -\varepsilon_1 \qquad (\varepsilon_1 < 1). \qquad (35)$$

To calculate the reflection efficiency, η, we have used the following method: As we have mentioned above, we start out from Eq. (4) describing the diffusion of pick-up ions in the velocity space while they are convected towards the shock by the solar wind. Instead of this equation we solve the system of SDEs (7a–c) as explained earlier in the text which describes the stochastic trajectories of a pick-up ions in the velocity space. When these ions arrive at the termination shock their local velocity, i.e. the actual magnitude v of their velocity in the solar wind rest frame is therefore known. Since the velocity distribution of the particles is considered to be isotropic, the associated pitch angles of these particles can simply be selected as a set of pseudo-random numbers with a uniform distribution. After that, equipped with the velocity space coordinates, by means of Eqs. (32a,b), (34a–c), and (35) we can conclude whether the particle will be reflected or transmitted downstream. Thus, by simulating a statistically relevant set of stochastic trajectories we obtain the reflection rates of pick-up ions at the termination shock.

From the conditions at the magnetohydrodynamic shock discontinuity one can obtain

$$\frac{B_2}{B_1} = \left[\cos^2\psi_1 + \left(\frac{\kappa - 1}{\kappa - 1/\sigma}\right)^2 \sin^2\psi_1\right]^{1/2} \qquad (36)$$

where

$$\kappa = \frac{1}{4\pi\rho_1}\left(\frac{B_1\cos\psi_1}{U_1\cos\delta_1}\right)^2, \quad \sigma = \rho_2/\rho_1. \qquad (37)$$

and can apply this to the solar wind flow near the termination shock using $\kappa \ll 1$. Then one can obtain from Eq. (36):

$$B_2/B_1 \approx (\cos^2\psi_1 + \sigma^2\sin^2\psi_1)^{1/2}$$

Within the model calculations of Baranov and Malama (1993) it is obtained that the compression ratio at the termination shock turns out to be $\sigma \approx 3.2$ (all over the termination shock!). So we have the value b which enters in the reflection conditions as a function of the shock normal angle.

The curves 1 in Figs. 9a,b,c give the number densities of the incident pick-up hydrogen at the upstream side of the termination shock for $\langle \tilde{B}_E^2 \rangle / B_E^2 = 0.2$ and for the shock normal angles $\psi_1 = 70°, 75°, 80°$, respectively. The curves 2 give the associated number densities of those incident particles for which the reflection conditions given by Eqs. (32a,b) are fulfilled. Finally the curves 3 give the number densities of reflected particles reappearing on the upstream side again measured in the solar wind frame. The curves 2 reveal that only the high-energy part of the pick-up ion distribution can undergo effective reflection at the shock. In addition one can see from a comparison of the curves 2 and 3 that the mean kinetic energy of the reflected particles has increased approximately by a factor of 10 due to the shock drift acceleration process. Thus it seems to be quite reasonable to consider the reflected pick-up ions with energy distributions given by the curves 3 as a seed population for ACRs. One can see also from inspection of Figs. 9a–c that the reflection efficiency at the termination shock strongly depends on the shock normal angle and hence on longitude. This dependence is shown in Fig. 10 which shows relative numbers, η, of reflected pick-up hydrogen and helium (η = flux of reflected pick-up ions/ flux of incident pick-up ions) as functions of longitude (off-axis angle) for various values of χ.

It is clear from Fig. 10 that the reflection rates of pick-up ions close to the nose part of the termination shock are very weak and increase with longitude. At the flanks of the termination shock the reflection rates reach their maximum magnitudes. As can be noticed in addition, a further important feature appearing in the results presented in Fig. 10 is the very strong dependence of the reflection efficiencies on the level of turbulence. Curves 1 show the reflection rates of the pick-up hydrogen and helium in the case when large-scale turbulence is ignored. From the comparison of these curves with the curves 2 and 3 calculated for the case of strong large-scale turbulence one can conclude that the reflection rates of pick-up ions at the termination shock and hence the injection efficiencies into ACRs are significantly determined by the level of large-scale oscillations in the solar wind. The differences in the reflection rates of the pick-up hydrogen and helium are connected with the differences in the acceleration efficiencies of these elements by solar wind turbulence as it can be seen from Fig. 6. Obviously this difference in the reflection rates can influence the relative abundances of the hydrogen and helium in ACRs.

It follows from our approach that the reflection rate of pick-up ions at the frontal part of the termination shock should vanish. This would be conclusive if one can consider only stationary magnetic fields near the shock and would not take into account the cross-shock potential. In fact large-scale magnetic fluctuations observed in the heliosphere can result in substantial deviations of the magnetic field from Parker's spiral (see Smith, 1993), so the termination shock in the frontal part will be a perpendicular one only for the average in time. In this case we can expect the reflection of pick-up ions in this region of the termination shock to occur according to results found by Liewer et al. (1993), or Kucharek & Scholer (1995). However, to describe this process of temporal changes correctly one should use a time-dependent theory of the interaction of large-scale fluctuations of the solar wind velocity and magnetic fields with the termination shock. This theory is not yet available in the literature. Another possibility for pick-ions to be reflected at the frontside of the termination shock is connected with surfings along the cross-shock potential wave (see Lee et al., 1996; Zank et al., 1996). However, this reflection mechanism can act only on particles having small velocities relative to the shock front. In fact both reflection mechanisms (reflection by the jump of the magnetic field at the shock and by the cross-shock potential) operate simultaneously: it first acts on high velocity particles, and second it acts on particles with small velocities in the shock rest frame. The relative efficiency of both mechanisms depends on the spectrum of incident particles (i.e. on the pre-acceleration by solar wind turbulence as we have shown here), on the shock normal angle, and on the structure of the shock transition.

We have considered in this section the interaction of charged particles with a shock front ignoring a possible influence of magnetic fluctuations on this process. It was, however, shown by numerical studies (see Decker 1988) that magnetic fluctuations at quasi-perpendicular shocks can result in a decrease of reflection efficiencies and in a broadening of energy distributions both for reflected and transmitted particles. These fluctuations also reduce the anisotropy of reflected particles near the shock.

6. Conclusions

In this paper we have studied by numerical methods the stochastic acceleration of the interplanetary pick-up hydrogen and helium by Alfvénic turbulence and by coherent large-scale oscillations both in the magnitudes of the solar wind velocity and magnetic field in the two-dimensional heliosphere. The large-scale oscillations describe travelling, forward and reverse shock waves closely connected with CIRs and MIRs. As can be deduced from numerous observations of deep spaceprobes the internal kinetic energy of these oscillations is comparable with the kinetic energy of the solar wind. Thus they can be considered as a powerful energy reservoir for particle accelerations in the heliosphere.

In the following we first compare calculated pick-up hydrogen spectra with observations made by ULYSSES during the passage of a CIR at 4.5 AU in October 1991 and can show that the observed spectrum can be explained sufficiently well by the stochastic acceleration of the pick-up hydrogen both by high-level Alfvénic and by large-scale turbulences simultaneously (Fig. 2). We show also that spectra of ions with the energies up to 400 KeV observed by VOYAGER 2 at 15 AU and 42 AU and negative radial gradients of these particles can naturally be explained within the framework of our present model of stochastic acceleration of pick-up ions (Fig. 3a,b). We predict considerable decrease in the magnitude of the radial gradients for the accelerated pick-up hydrogen at distances $r \geq 60$ AU (Fig. 4). Our results demonstrate the dominant role of large-scale turbulences in the formation of the high-energy tail of pick-up ions in the outer heliosphere (up to several hundred KeV/nuc) while the small-scale Alfvénic turbulences essentially provides the initial "pre-heating" of particles up to several KeV/nuc.

The pre-acceleration of pick-up ions by solar wind turbulence is a necessary step for invoking their further acceleration up to ACR energies at the termination shock, since we have shown that only those pick-up ions with sufficiently high energies are reflected at the termination shock with its abrupt changes in both the strength and the direction of the magnetic field. The mean energy of reflected particles increases by approximately a factor of 10 (Fig. 10). We have obtained the very strong dependence of the reflection rates on longitude in the ecliptic plane because of the specific prolate shape of the termination shock. Thus we expect that the maximum injection of pick-up ions into ACRs takes place at the flanks of the shock. Besides that, since the magnitude of large-scale oscillations of the solar wind velocity and magnetic field decreases with latitude the acceleration rate of pick-up ions in the solar wind and hence the reflection rate at the termination shock must also decrease with latitude.

Interestingly enough it follows from our present calculations that the pick-up helium is more efficiently accelerated by solar wind turbulences than pick-up hydrogen. This can be seen in Fig. 6 where the relative overabundance of helium at high energies (i.e. at 100 to 400 KeV/nuc) is evident. Consequently also the reflection rate of the accelerated pick-up helium at the termination shock exceeds that of pick-up hydrogen (Fig. 11) meaning that the helium pick-up ion injection rate into ACR helium is strongly enhanced compared to the injection into ACR hydrogen. These two latter conclusions differ from our earlier results (Chalov et al., 1995; Chalov & Fahr, 1996) in which only the effect of Alfvénic turbulences had been taken into account. Also in the hybrid simulation calculations of pick-up ion accelerations in the termination shock structure by Kucharek and Scholer (1995) this preference of pick-up helium reflection rates over pick-up hydrogen was not obtained. To the opposite the hydrogen reflection rates were obtained as higher by factors 5 to 10. This astonishing difference found in the present paper is explained by the inclusion of the effect of solar wind large-scale oscillations on pick-up ion acceleration in our present model, while in the paper by Chalov & Fahr (1996) only the stochastic acceleration of pick-up ions by small scale Alfvénic turbulence had been considered which more efficiently acts on hydrogen rather than on helium.

The comparable reflection rates obtained for pick-up hydrogen in the present paper and in the earlier paper where only Alfvénic turbulence has been considered is explained by the different spatial behavior of the correlation lengths of Alfvénic turbulence assumed in these two papers. Here we assume that this correlation length increases linearly with the distance from the Sun (i.e. $\delta = 1$ taken in Eq. (16)), while in Chalov & Fahr (1996) instead we had adopted that the correlation length is constant throughout the heliosphere. Of course, in the first case, relative to the latter, the role of Alfvénic turbulence in accelerating pick-up ions is increased in the outer heliosphere.

Our conclusion concerning favourable reflection conditions for pick-up helium compared to pick-up hydrogen at the termination shock is of obvious importance for an explanation of the observed relative deficit of the hydrogen in ACRs with respect to ACR helium (see e.g. Cummings & Stone 1995; Stone, Cummings and Webber, 1996). At equal injection conditions these authors would claim for interplanetary helium densities enhanced by a factor of 7 to 10 with respect to interplanetary hydrogen densities. However, under the auspices presented in this paper here one can simply ascribe these anomalies to the fact that pick-up helium undergoes a more efficient injection into the ACR regime.

At the end of this paper we would like to mention that the results of this paper showing ongoing accelerations of pick-up ions up to large solar distances, especially also by CIR's, clearly prove that extrapolations as done by Liewer et al. (1995) of energetic particle spectra obtained by VOYAGER-1 and VOYAGER-2 at 15 AU and 21 AU to large distances, only thereby taking into account adiabatic deceleration processes, are unjustified. The spectra instead are clearly processed to higher energies on the passage of pick-ups to large distances.

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