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1. Theoretical Framework

The obliquity of the shock at the time of the crossing by Voyager 2 was obtained through the Rankine-Hugoniot conditions. The upstream conditions at the Voyager 2 location are: $\hat{V}_1 = V_1\hat{R} - V_S\hat{n}$ and $\hat{B}_1 = B_1\hat{T}$, where $\hat{R}$ and $\hat{T}$ are in the RTN coordinate system and V_S is the shock speed. Considering a shock coordinate system where the parallel axis || is along the normal of the shock, $\hat{n} = \cos(\theta_1)\hat{R} + \sin(\theta_1)\cos(\phi_1)\hat{T} + \sin(\theta_1)\sin(\phi_1)\hat{N}$, where θ_1 and ϕ_1 are the angles between the normal **n** and the radial upstream velocity V_1. θ_1 is defined as the angle measured between **n** and **R.** ϕ_1 is the angle between the projection of n on the (T, N) plan and T (see Supplementary figure 1). The perpendicular axis are taken as $\hat{n}_{\perp 1} = \hat{T}\times\hat{n} / \left\|\hat{T}\times\hat{n}\right\|$, and $\hat{n}_{\perp 2} = \hat{n}\times\hat{n}_{\perp 1} / \left\|\hat{n}\times\hat{n}_{\perp 1}\right\|$.

The three-dimensional Rankine-Hugoniot conditions in this shock coordinate system (after a transformation to the deHoffman -Teller frame) are:

$$\mathrm{v}_{2\parallel} = \frac{\mathrm{v}_{1\parallel}}{r}$$

$$\mathrm{v}_{2\perp 1} = \mathrm{v}_{1\perp 1} \qquad \text{Eqs. (1-3)}$$

$$\mathrm{v}_{2\perp 2} = \mathrm{v}_{1\perp 2} + \frac{B_{1\perp 2}}{B_{1\parallel}}\frac{(r-1)\mathrm{v}_{1\parallel}\mathrm{v}_{A1\parallel}^2}{\left(\mathrm{v}_{1\parallel}^2 - r\mathrm{v}_{A1\parallel}^2\right)}$$

where the index "1" and "2" refer respectively, to the upstream and downstream quantities; V_{A1} is the upstream Alfven speed. In the RTN frame Eqs. (1-3) become:

$$\mathrm{V}_{2R} = \left(\frac{\mathrm{V}_1\cos(\theta_1) - \mathrm{V}_S}{\mathrm{r}}\right)\cos(\theta_1) + \frac{V_1\sin^2(\theta_1)\sin^2(\phi_1)}{\alpha_1^2}$$
$$-\left(-\frac{V_1\cos(\theta_1)\sin(\theta_1)\cos(\phi_1)}{\alpha_1} + \tan(\chi_1)\frac{(r-1)\left(V_1\cos(\theta_1) - V_S\right)\left(V_{A1}\cos(\chi_1)\right)^2}{\left(\left(V_1\cos(\theta_1) - V_S\right)^2 - r\left(V_{A1}\cos(\chi_1)\right)^2\right)}\right)\frac{\cos(\theta_1)\sin(\theta_1)\cos(\phi_1)}{\alpha_1}$$

$$\mathrm{V}_{2T} = \left(\frac{\mathrm{V}_1\cos(\theta_1) - \mathrm{V}_S}{\mathrm{r}}\right)\sin(\theta_1)\sin(\phi_1) +$$
$$+\left(-\frac{V_1\cos(\theta_1)\sin(\theta_1)\cos(\phi_1)}{\alpha_1} + \tan(\chi_1)\frac{(r-1)\left(V_1\cos(\theta_1) - V_S\right)\left(V_{A1}\cos(\chi_1)\right)^2}{\left(\left(V_1\cos(\theta_1) - V_S\right)^2 - r\left(V_{A1}\cos(\chi_1)\right)^2\right)}\right)\alpha_1 \qquad \text{Eqs.(4-6)}$$

$$V_{2N} = \left(\frac{V_1\cos(\theta_1) - V_S}{r}\right)\sin(\theta_1)\sin(\phi_1) - \frac{V_1\cos(\theta_1)\sin(\theta_1)\sin(\phi_1)}{\alpha_1^2}$$

$$-\left(-\frac{V_1\cos(\theta_1)\sin(\theta_1)\cos(\phi_1)}{\alpha_1} + \tan(\chi_1)\frac{(r-1)(V_1\cos(\theta_1) - V_S)(V_{A1}\cos(\chi_1))^2}{\left((V_1\cos(\theta_1) - V_S)^2 - r(V_{A1}\cos(\chi_1))^2\right)}\right)\frac{\sin^2(\theta_1)\cos(\phi_1)\sin(\phi_1)}{\alpha_1}$$

$\alpha_1 = \sqrt{\sin^2(\theta_1)\sin^2(\phi_1) + \cos^2(\theta_1)}$, χ_1 is the angle between B_1 and the shock normal: $\cos(\chi_1) = \sin(\theta_1)\cos(\phi_1)$ and r is the compression ratio, $r=\rho_2/\rho_1$.

We considered the Rankine-Hugoniot conditions at the location of Voyager 2. Although there were three shock crossings by V2, we are interested in the average upstream conditions at the termination shock along V2 trajectory. The average upstream conditions can be taken as V_1=340km/s; B_1=0.11μG; n_p=0.0018cm^{-3} which gives an upstream Alfven speed V_{1A}=56.5km/s. We took the compression ratio as r=2.0. Using Eqs. (4-6 of SI) and equating to the average values measured downstream of V_{2R}=138 km/s; V_{2T}=48 km/s and V_{2N}=-14km/s, we obtained the obliquity of the termination shock at the Voyager 2 crossing time.

Below we give more information about our three dimensional magnetohydrodynamical (MHD) model with 5-fluids. Our model uses a multifluid approach to describe the neutral H atoms. Our three dimensional MHD model is a five fluid model (similar to Alexashov & Izmodenov[24] and Zank et al.[25]) and was benchmarked with kinetic models[12,13]. In the multi-fluid approach there are four populations of neutral H atoms, for every region in the interaction between the solar wind and the interstellar wind. Population 4 represents the H atoms of interstellar origin. Population 1 represents the H atoms that exist in the region between the bow shock and heliopause. Populations 3 and 2 represent the H atoms in the supersonic solar wind and in the compressed region between the termination shock and the heliopause, respectively. All four H populations are described by separate systems of the Euler equations with the corresponding source terms. The ionized component interacts with the H neutrals via charge exchange, described in our model by the source terms derived in McNutt et al.[26] with the cross section as given by Lindsay and Stebbings[12].

The parameters for the density, velocity and temperature for the ions and neutrals in the interstellar medium reflect the best observational values and are given in Supplementary Table 1. We used fixed inner boundary conditions for the ionized fluid and soft boundaries for the neutral fluids. The outer boundaries were all outflows with the exception of the -x boundary, where the inflow conditions were imposed for the ionized and the population of neutrals coming from the interstellar medium (Population 4). The grid has the inner boundary at 30 AU and the outer boundaries are set at -1500 AU and 1500 AU in x, y, z directions, respectively. The computational cell size ranges from 0.73AU to 93.7AU.

2. Supplementary Table 1: Model parameters of the three dimensional MHD 5 fluids model.

Solar Wind: Inner Boundary (30AU)	*Interstellar Wind*	*H Neutral ISW*
$N=8.74\times10^{-3}$ cm^{-3}	$N=0.06cm^{-3}$	$N=0.18cm^{-3}$
Parker Spiral with B=2μG at the solar equator	B_{ISM}=2.5μG -4.4μG	
V=417.07km/s	V=26.3km/s	V=26.3km/s
$T=1.087\times10^{5}K$	T=6519K	T=6519K

3. Discussion

We find that the effect of the grid resolution is small, less than 8°. To estimate the effect of the grid resolution, we ran three different refinement levels at the termination shock for the case where B_{ISM} has the orientation of $\beta=60°$, $\alpha=20°$ and intensity of 4.4μG. The refinement levels chosen were low, medium and high with grid resolutions of 3 AU, 1.5 AU and 0.75 AU respectively.

As the resolution increases, the gradient of the radial velocity steepens as expected (see Supplementary Fig. 2d). Each point in the curve represents an average value from a set of points in the three-dimensional grid located within a radius of 6AU from the Voyager 2 trajectory. We chose 6AU since this corresponds to two grid points of the low-resolution case. The error bars in panel care estimated as the root-mean-square of the all the points. We estimate θ_1 and ϕ_1 by averaging their values from upstream to downstream of the termination shock. The average value for θ_1 and ϕ_1 changes, respectively by 5° and 19° as you go from the low to the high refinement cases. The average values of θ between upstream and downstream of the low, medium and high resolution are respectively, -51.2±6.7°, -62.0±5.75° and -53.6±3.6°.

We estimated as well the dependence of θ (the orientation of the flow which in turn constrains the orientation of the magnetic field) on the shock velocity, finding it to be less than 0.1°. We estimated θ_1 and ϕ_1 in the Rankine-Hugoniot equations setting the shock velocity to zero. The values obtained were $\theta_1=15°$ and $\phi_1=165°$, which give θ of -15.9°, less than 0.1° difference from the observed value. The values obtained for V_{2T} and V_{2N} are slightly different than the ones measured by Voyager 2, V_{2N}=38.8km/s; V_{2T}=-11km/s, but the ratio V_{2N}/V_{2T} remains the same. The velocity component that is more affected by the shock speed is V_{2R}=181km/s. Therefore, we expect that our stationary models give comparable angles θ but higher values of V_{2R} than models with a moving shock ($V_s\neq0$). (For reference, hydrodynamic model with $V_A=0$ and with $V_s\neq0$ gives V_{2R}=139 km/s; V_{2T}=52 km/s and V_{2N}=-14km/s with θ=-15°.)

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4. Supplementary Figures and Legends

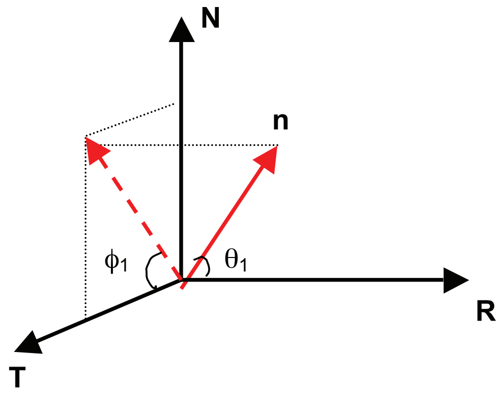

Figure S1: Definition of the obliquity angles of the shock in the RTN coordinate system. θ_1 is defined as the angle measured between **n** and **R.** ϕ_1 is the angle between the projection of n on the (T, N) plane and T.

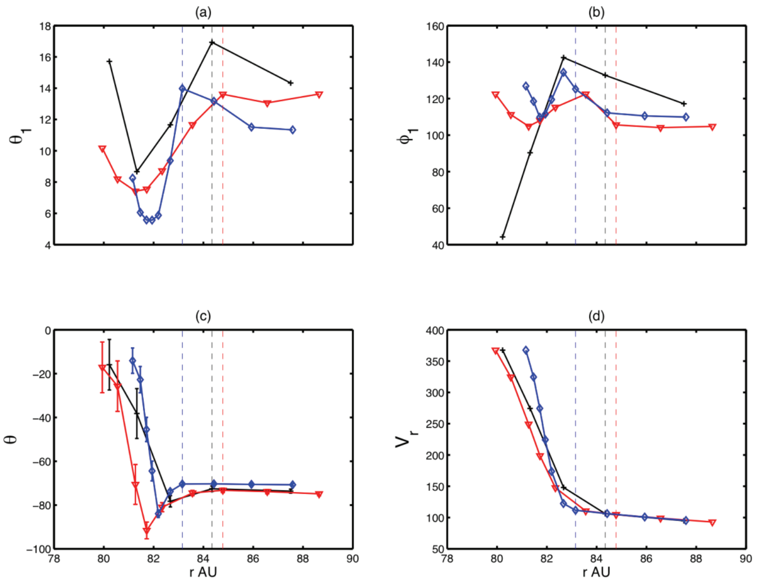

Figure S2: Effect of resolution on the shock. The angles θ_1, ϕ_1, θ and radial velocity Vr are between upstream and downstream of the termination shock for three different refinement levels. The three refinement levels are: low (black curve), medium (red) and high (blue) for the case where B_{ISM} has the orientation of β=60°, α=20°and B=4.4μG. In panels (c) and (d) the downstream locations for the three cases are indicated by the dashed lines. We took the downstream location as the position where the radial velocity has the strongest inflection. The downstream locations for the three cases are indicated by the dashed lines.

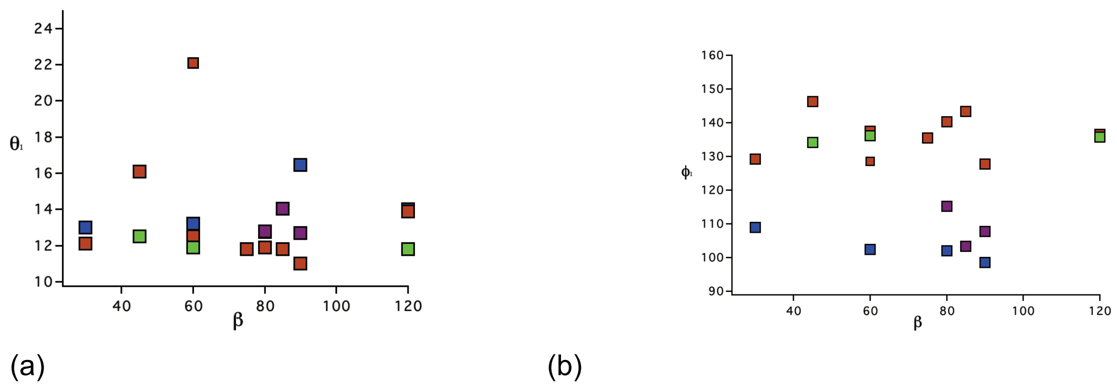

Figure S3: The termination shock obliquity as a function of the B_{ISM} orientation. Panels (a) and (b) show respectively, the angle θ_1 and ϕ_1 as a function of angle β from and α. The orientations with α=20°, 30°, 45° and 60° are shown in blue, magenta, brown, and green, respectively. The size of the squares corresponds to the intensity of B_{ISM} from 2.5-5.5μG. We estimate θ_1 and ϕ_1 by averaging their values from upstream to downstream of the termination shock.

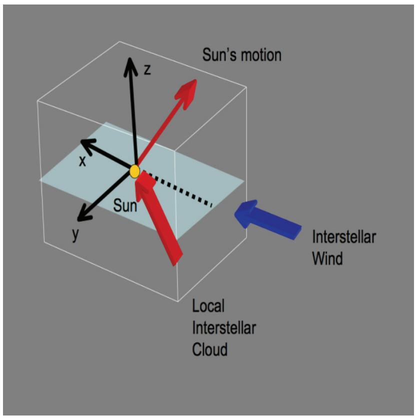

Figure S4: The motion of the Sun in the Local Standard of Rest (LSR) and our model. The cloud of material that surrounds the solar system is called the Local Interstellar Cloud. In the LSR the motions of the sun through space and the Local Interstellar Cloud are seen to be nearly perpendicular (shown in red arrows). The result of these two motions is that we observe interstellar material flowing toward the sun at about 26 kilometers per second, called the Interstellar Wind from a direction close to the plane of the ecliptic (shown by the blue arrow). We use the interstellar wind direction to define the x-axis of our model.